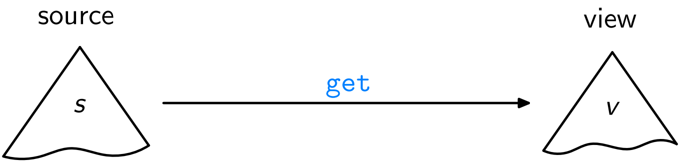
Semantic Bidirectionalization and the Constant-Complement Perspective

Janis Voigtländer

University of Bonn

BT-in-ABC'10

Bidirectional Transformation



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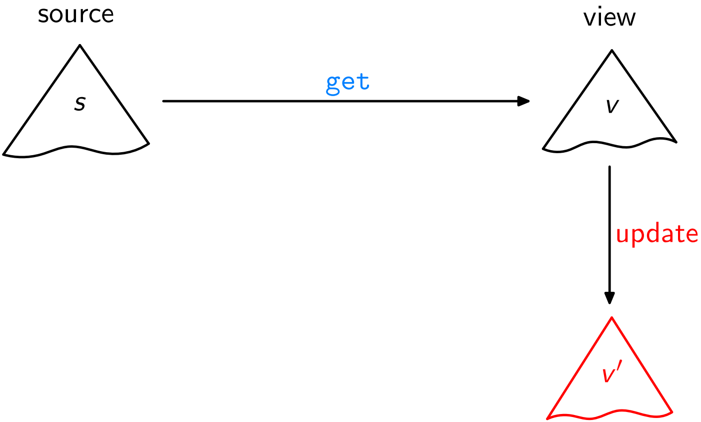

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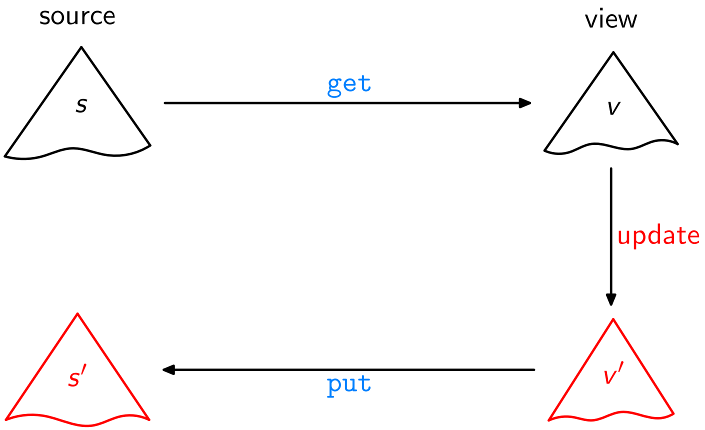

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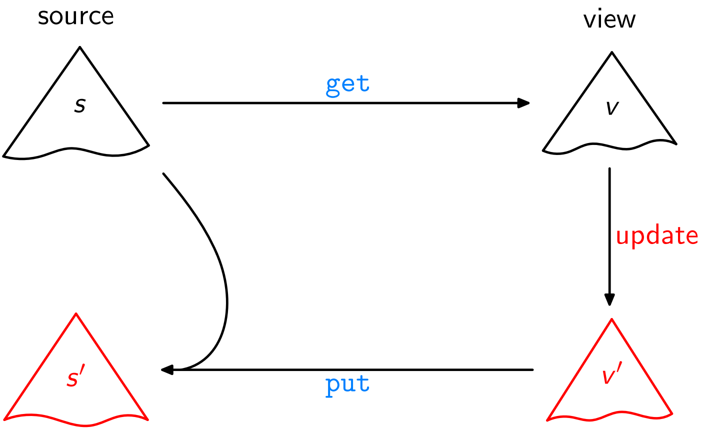

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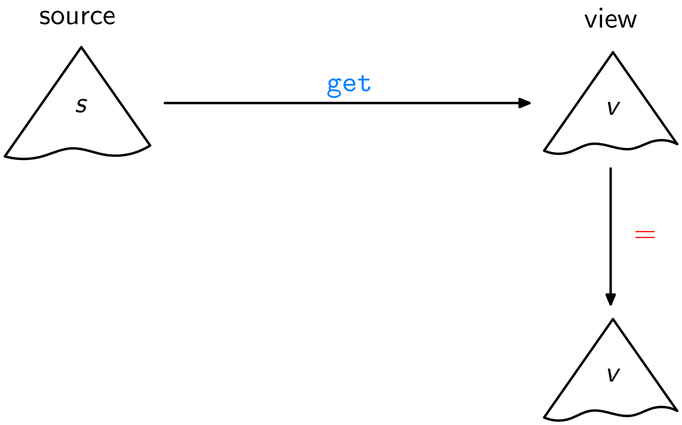
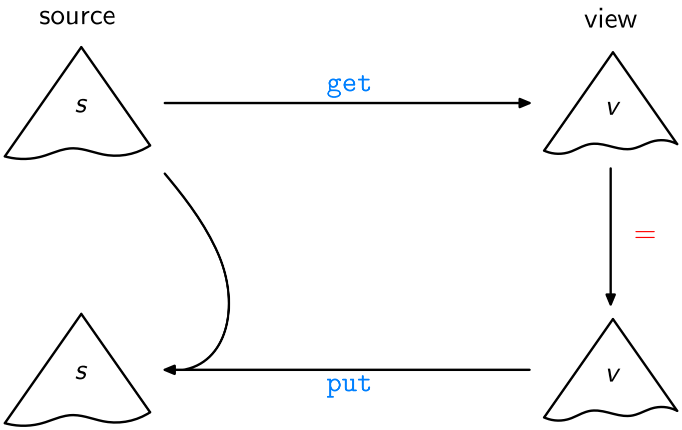


Acceptability / GetPut

Bidirectional Transformation



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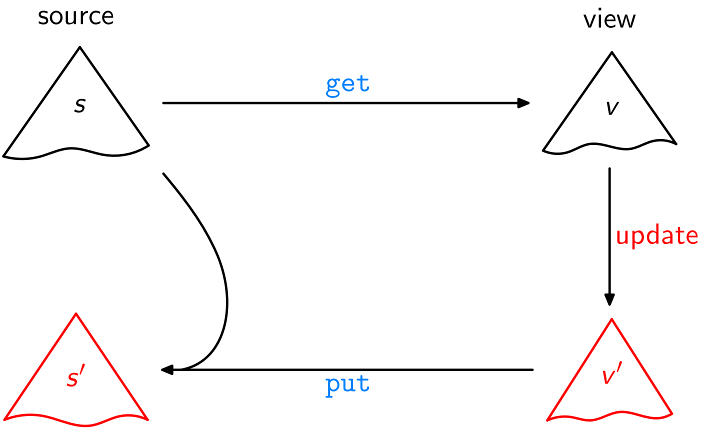

Consistency / PutGet

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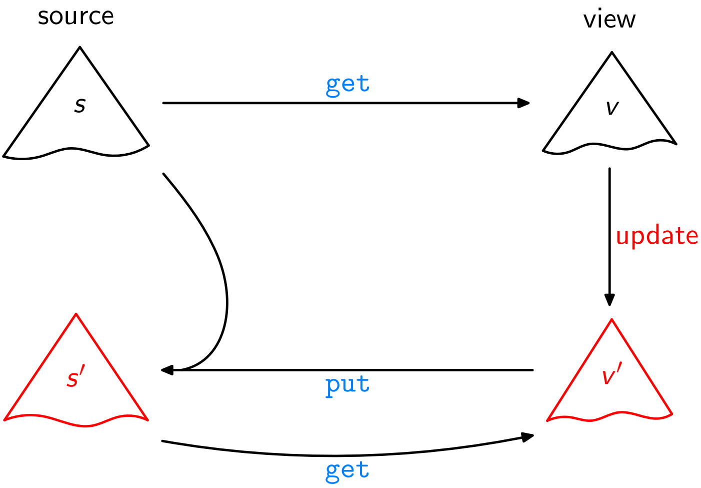

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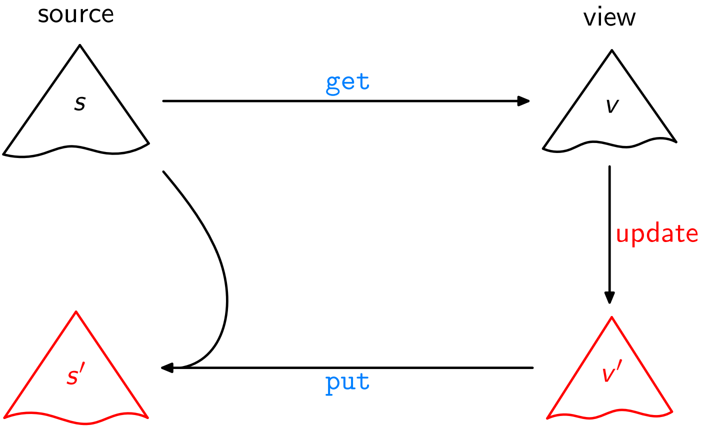

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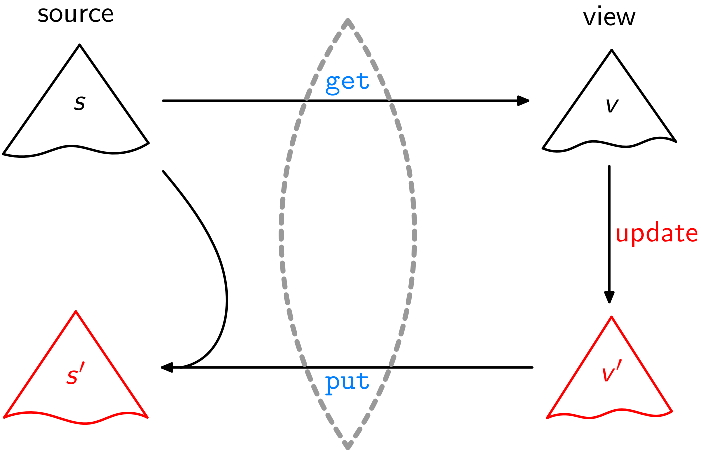
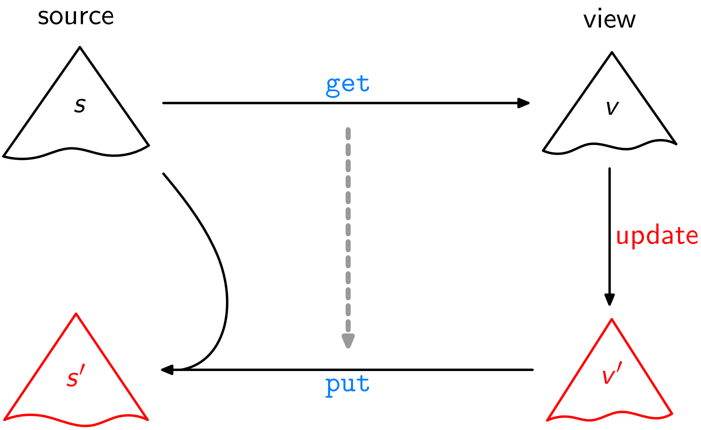


Lenses, DSLs

[Foster et al., ACM TOPLAS'07, ...]

Bidirectional Transformation



Bidirectionalization

[Matsuda et al., ICFP'07]

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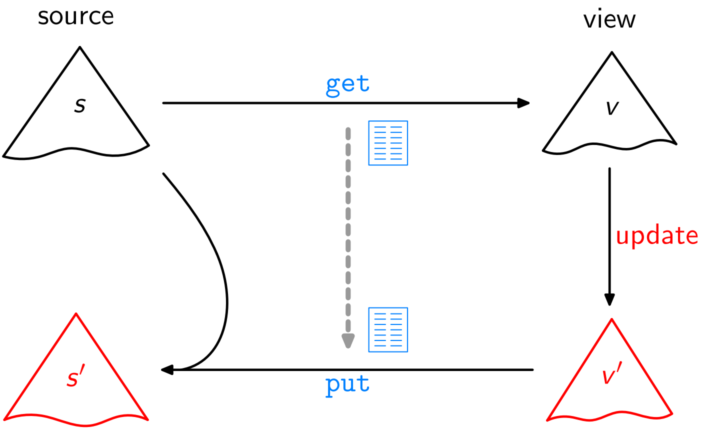
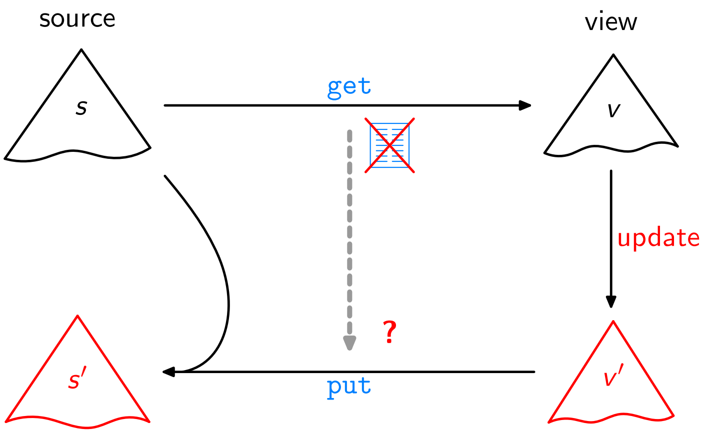


Syntactic Bidirectionalization

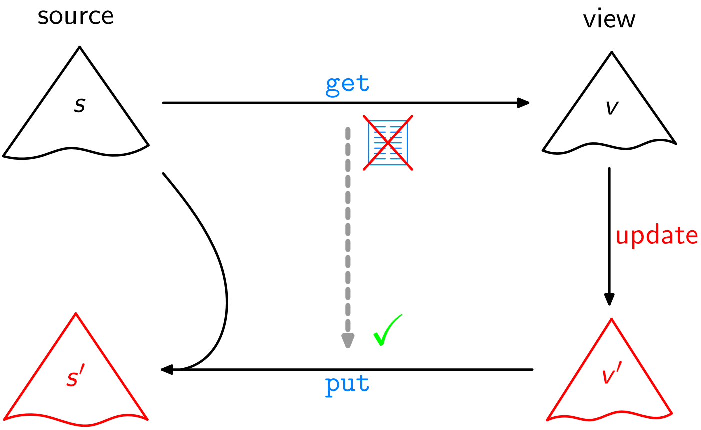
[Matsuda et al., ICFP'07]

Bidirectional Transformation



Semantic Bidirectionalization

Bidirectional Transformation



Semantic Bidirectionalization

[V., POPL'09]

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Aim: Write a higher-order function `bff` such that any `get` and `bff get` satisfy GetPut, PutGet, ...

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Examples:

$$\text{“abc”} \xrightarrow{\quad\texttt{tail}\quad} \text{“bc”}$$

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$$\begin{tikzcd}
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& & \text{“de”}
\end{tikzcd}$$

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\text{``ade''} & & & \text{``de''} \arrow[lll, "\texttt{bff tail}"]
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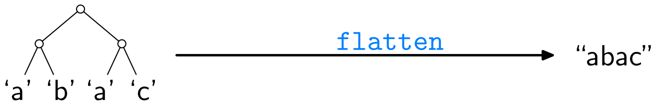

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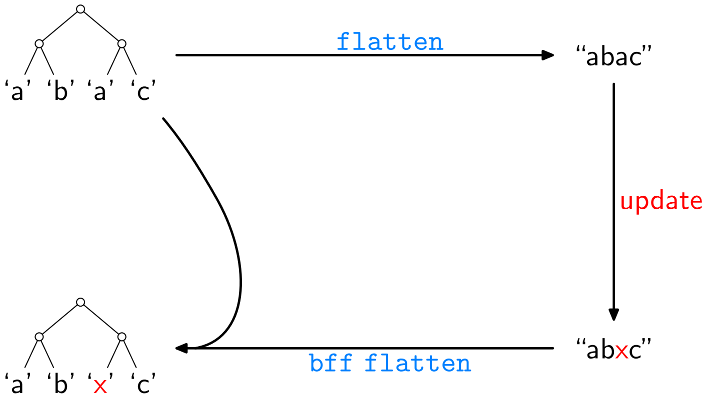

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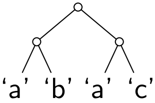

$\xrightarrow{\texttt{nub} \circ \texttt{flatten}}$ "abc"

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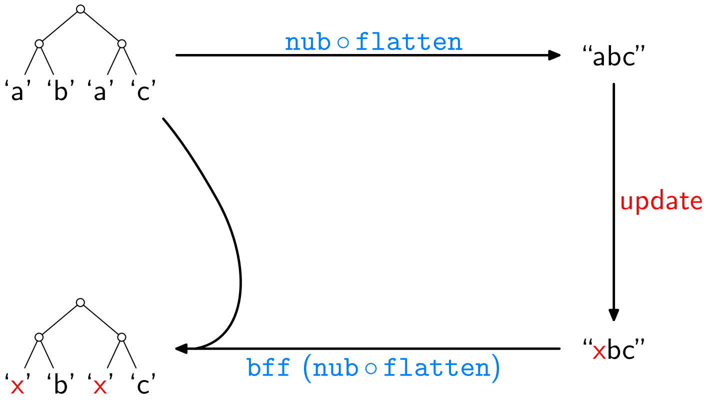

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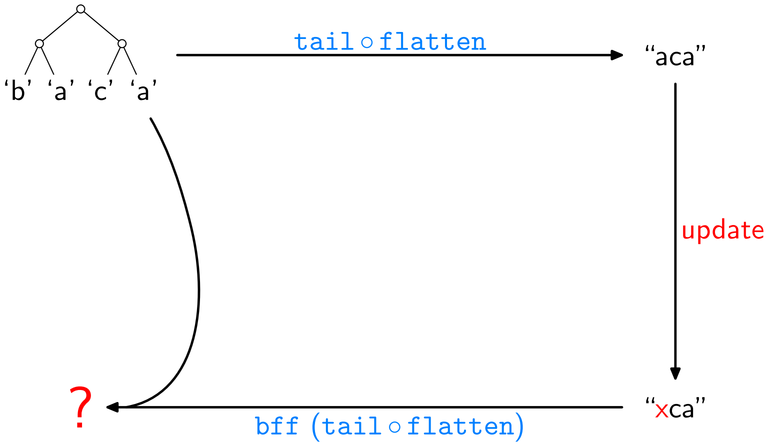
Overview of the Bidirectionalization Method



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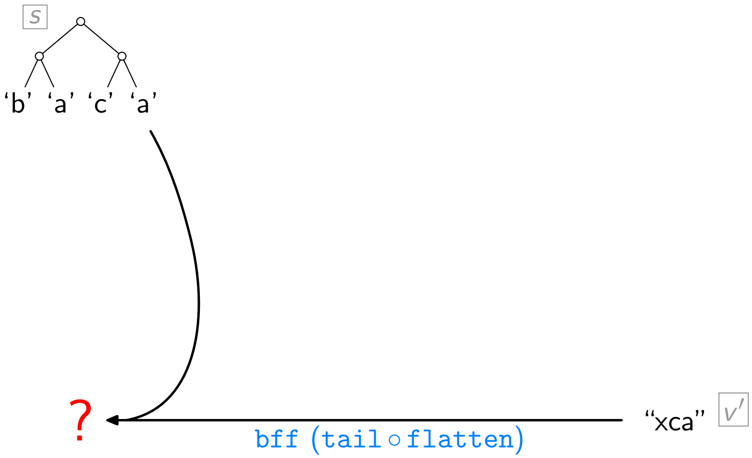

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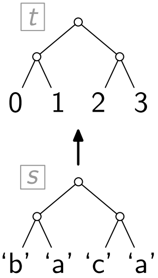

"xca" $\boxed{v'}$

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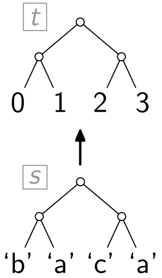

0	$\rightarrow$	‘b’
1	$\rightarrow$	‘a’
2	$\rightarrow$	‘c’
3	$\rightarrow$	‘a’

g

“xca” v'

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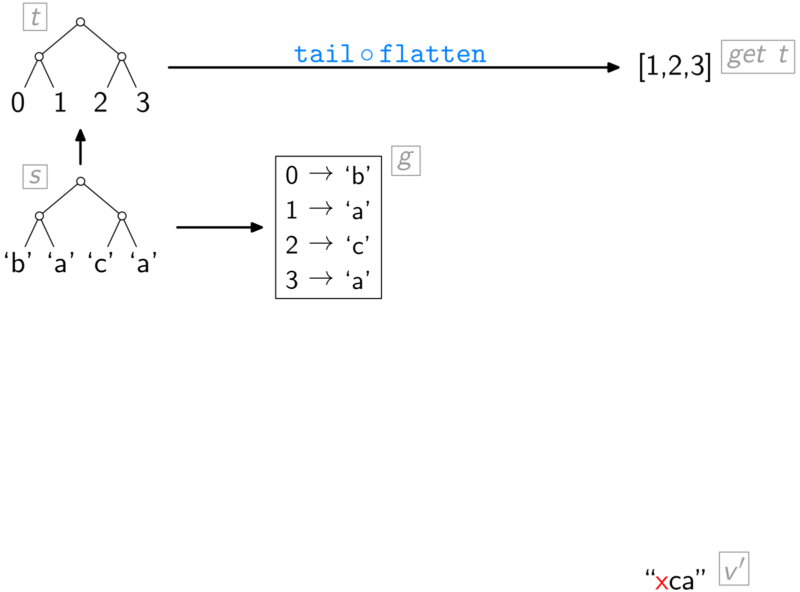

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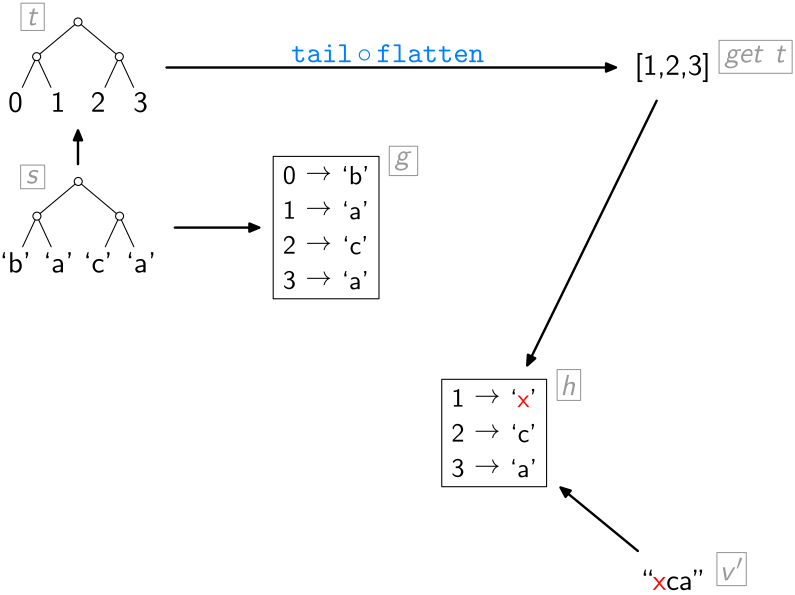

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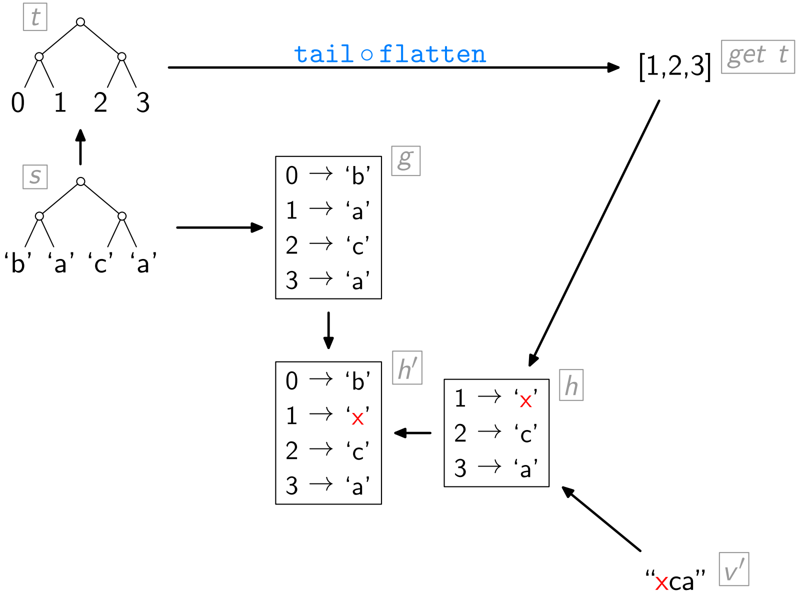

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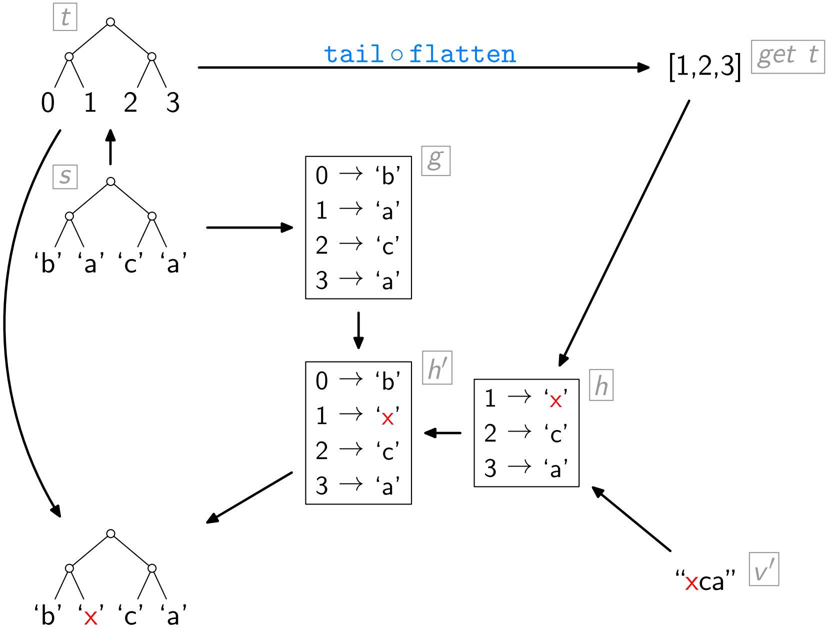

The Constant-Complement Approach [Bancilhon & Spyratos, ACM TODS'81]

In general, given

$$\texttt{get} :: S \to V$$

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Important: `compl` should "collapse" as much as possible.

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For the moment, be maximally conservative.

The Complement Function

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\begin{aligned}
&\texttt{compl} :: [\alpha] \to (\mathsf{Int}, [\alpha]) \\
&\texttt{compl}\ s = \textbf{let}\ n \ = (\texttt{length}\ s) - 1 \\
&\qquad\qquad\quad\ \ t \ = [0..n] \\
&\qquad\qquad\quad\ \ g \ = \texttt{zip}\ t\ s \\
&\qquad\qquad\quad\ \ g' = \texttt{filter}\ (\lambda(i, \_) \to \texttt{notElem}\ i\ (\texttt{get}\ t))\ g \\
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$$\texttt{get} = \texttt{reverse} \quad \rightsquigarrow \quad \texttt{compl}\ \text{“abcde”} = (5, [\,])$$

An Inverse of $\lambda s \rightarrow (\texttt{get}\ s, \texttt{compl}\ s)$

$$
\begin{array}{l}
\texttt{inv} :: ([\alpha], (\mathsf{Int}, [\alpha])) \rightarrow [\alpha] \\
\texttt{inv}\ ([\,], (0, \_)) = [\,] \\
\texttt{inv}\ (v', (n+1, as)) = \\
\quad \textbf{let}\ t \ \ = [0..n] \\
\quad\quad\ \ h \ \ = \texttt{assoc}\ (\texttt{get}\ t)\ v' \\
\quad\quad\ \ g' = \texttt{zip}\ (\texttt{filter}\ (\lambda i \rightarrow \texttt{notElem}\ i\ (\texttt{get}\ t))\ t)\ as \\
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$$\mathtt{get} = \mathtt{take}\ 3 \quad \rightsquigarrow \quad \mathtt{inv}\ (\text{``xyz''}, (5, [\text{'d'}, \text{'e'}])) = \text{``xyzde''}$$

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Correctness

To prove formally:

- $\texttt{inv}\ (\texttt{get}\ s, \texttt{compl}\ s) = s$
- if $\texttt{inv}\ (v, c)$ defined, then $\texttt{get}\ (\texttt{inv}\ (v, c)) = v$
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we have, for arbitrary f and l,

$$\texttt{map}\ f\ (\texttt{get}\ l) \quad = \quad \texttt{get}\ (\texttt{map}\ f\ \ l)\,.$$

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Given an arbitrary list s of length $n + 1$, set $l = [0..n]$, $f = (s\,!!)$, leading to:

$$\texttt{map}\ (s\,!!)\ (\texttt{get}\ [0..n]) \quad = \quad \texttt{get}\ (\texttt{map}\ (s\,!!)\ [0..n])$$

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Use a free theorem [Wadler, FPCA'89], namely that for every

$$\texttt{get} :: [\alpha] \to [\alpha]$$

we have, for arbitrary f and l,

$$\texttt{map}\ f\ (\texttt{get}\ l) \quad = \quad \texttt{get}\ (\texttt{map}\ f\ \ l)\,.$$

Given an arbitrary list s of length $n+1$, set $l = [0..n]$, $f = (s\,!!)$, leading to:

$$\begin{aligned} \texttt{map}\ (s\,!!)\ (\texttt{get}\ [0..n]) \quad &= \quad \texttt{get}\ (\underbrace{\texttt{map}\ (s\,!!)\ [0..n]}_{s}) \\ &= \quad \texttt{get}\ \ s \end{aligned}$$

Correctness

To prove formally:

- $\mathtt{inv}\ (\mathtt{get}\ s, \mathtt{compl}\ s) = s$
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Altogether, So Far:

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\begin{aligned}
&\texttt{compl} :: [\alpha] \to (\mathsf{Int}, [\alpha])\\
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&\quad \textbf{let}\ t\ \ = [0..n]\\
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“Fusion”

Inlining `compl` and `inv` into `put`, plus some clever rewriting:

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\begin{array}{l}
\texttt{put}\ [\,]\ [\,] = [\,] \\
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\qquad\qquad\qquad\qquad\qquad\qquad\quad \begin{array}{ll}
\text{Nothing} & \rightarrow (i, b) : m \\
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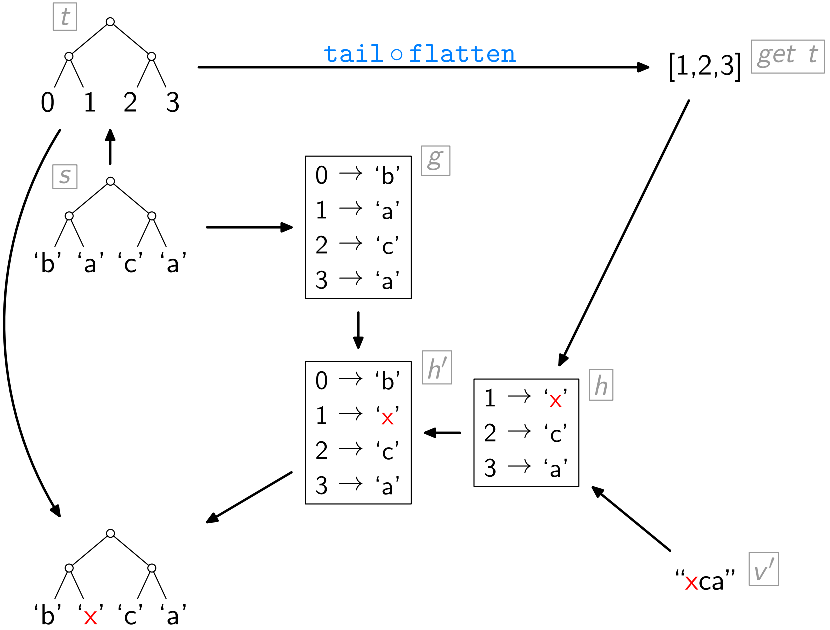
Actual code only slightly more elaborate!

Overview of the Bidirectionalization Method



Extending the Technique

Major Problem:

- Shape-affecting updates lead to failure.

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- Being maximally conservative this way often does not "collapse enough".
- For example:

  $\texttt{get} = \texttt{tail} \quad \rightsquigarrow \quad$ `put` "abcde" "xyz" fails precisely because `compl` "abcde" = (5, ['a'])

Assuming Shape-Injectivity

So assume there is a function

$$\texttt{shapeInv} :: \mathsf{Int} \rightarrow \mathsf{Int}$$

with, for every source list s,

$$\texttt{length}\ s = \texttt{shapeInv}\ (\texttt{length}\ (\texttt{get}\ s))$$

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But how to obtain `shapeInv` ???

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But how to obtain $\texttt{shapeInv}$???

Just for experimentation:

$$
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\texttt{shapeInv} :: \mathsf{Int} \to \mathsf{Int} \\
\texttt{shapeInv}\ l_v = \texttt{head}\ [n + 1 \mid n \leftarrow [0..],\ (\texttt{length}\ (\texttt{get}\ [0..n])) == l_v]
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$$

Some Tests

Works quite nicely in some cases:

$$\texttt{get} = \texttt{tail} \quad \rightsquigarrow \quad \begin{array}{l} \texttt{put}\ \text{``abcde''}\ \text{``xyz''} = \text{``axyz''}, \text{ using} \\ \texttt{compl}\ \text{``abcde''} = [\text{'a'}] \end{array}$$

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But not so in others:

$$\texttt{get} = \texttt{take}\ 3 \quad \rightsquigarrow \quad \texttt{put}\ \text{“abcde”}\ \text{“abc”} = \text{“abc”}$$

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Conclusion

[V., POPL'09]:

- very lightweight, easy access to bidirectionality
- full treatment of equality and ordering constraints
- proofs, using free theorems and equational reasoning
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Outlook:

- ... could also be a way to inject/exploit "user knowledge"
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- efficiency issues untackled so far, ...

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