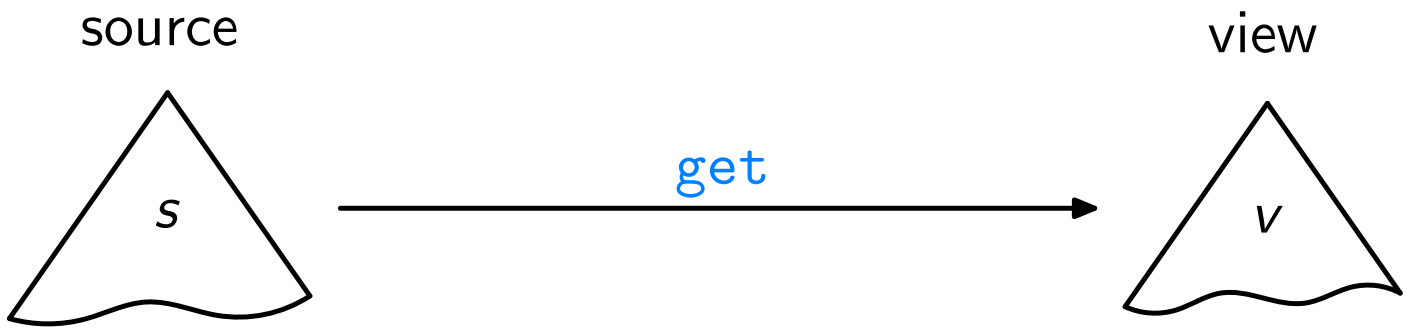
Semantic Bidirectionalization and the Constant-Complement Perspective

Janis Voigtländer

University of Bonn

BT-in-ABC'10

Bidirectional Transformation



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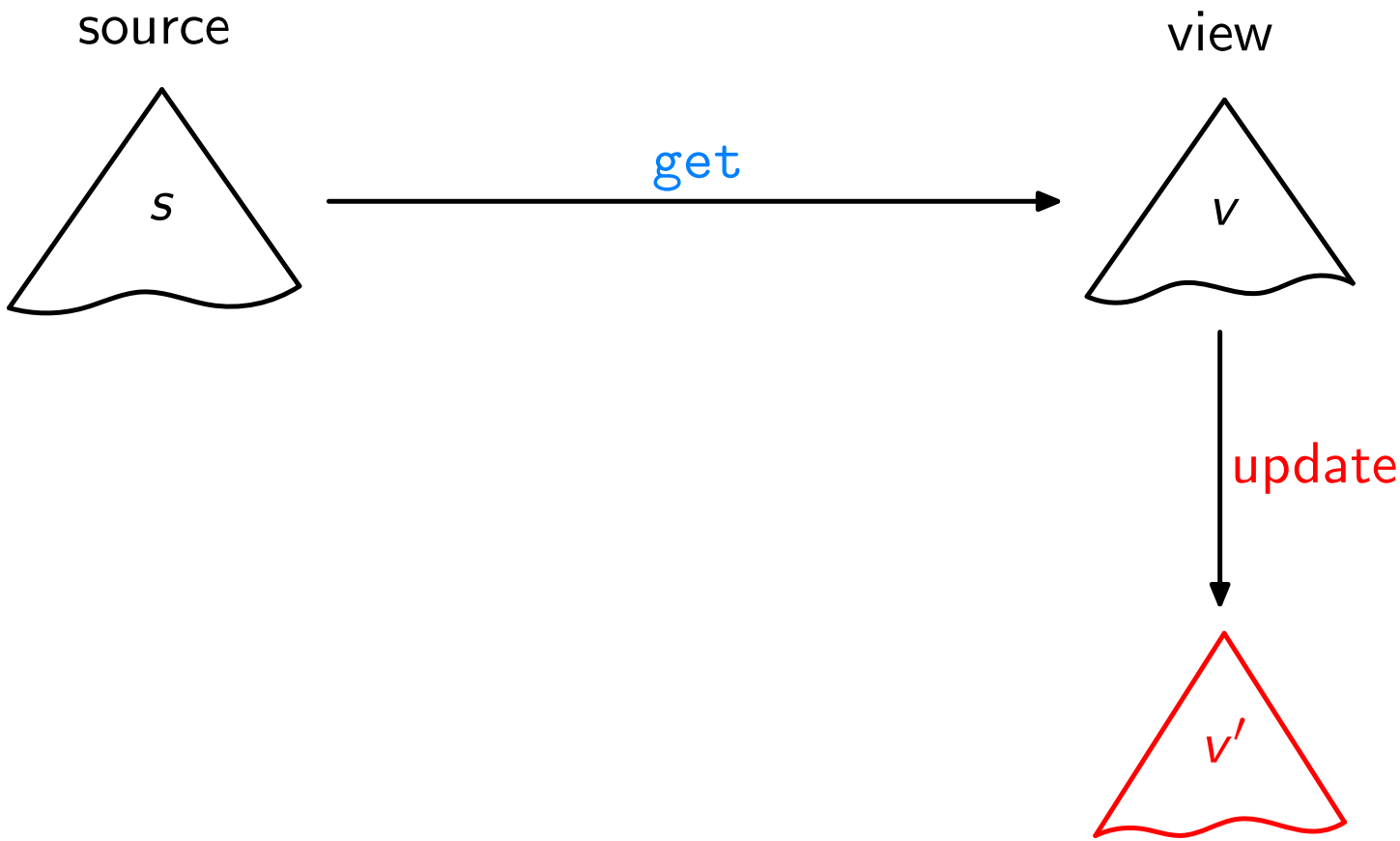

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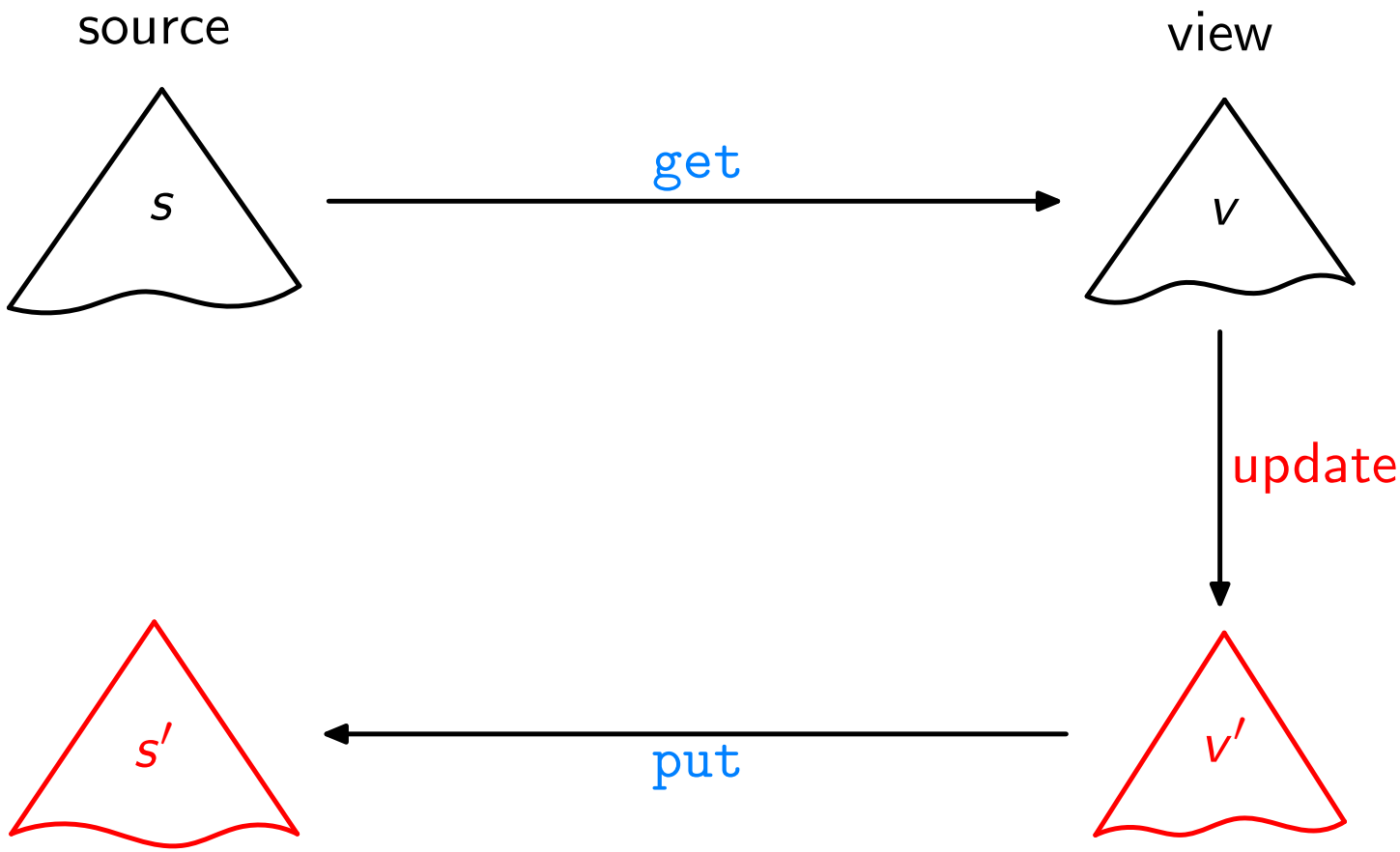

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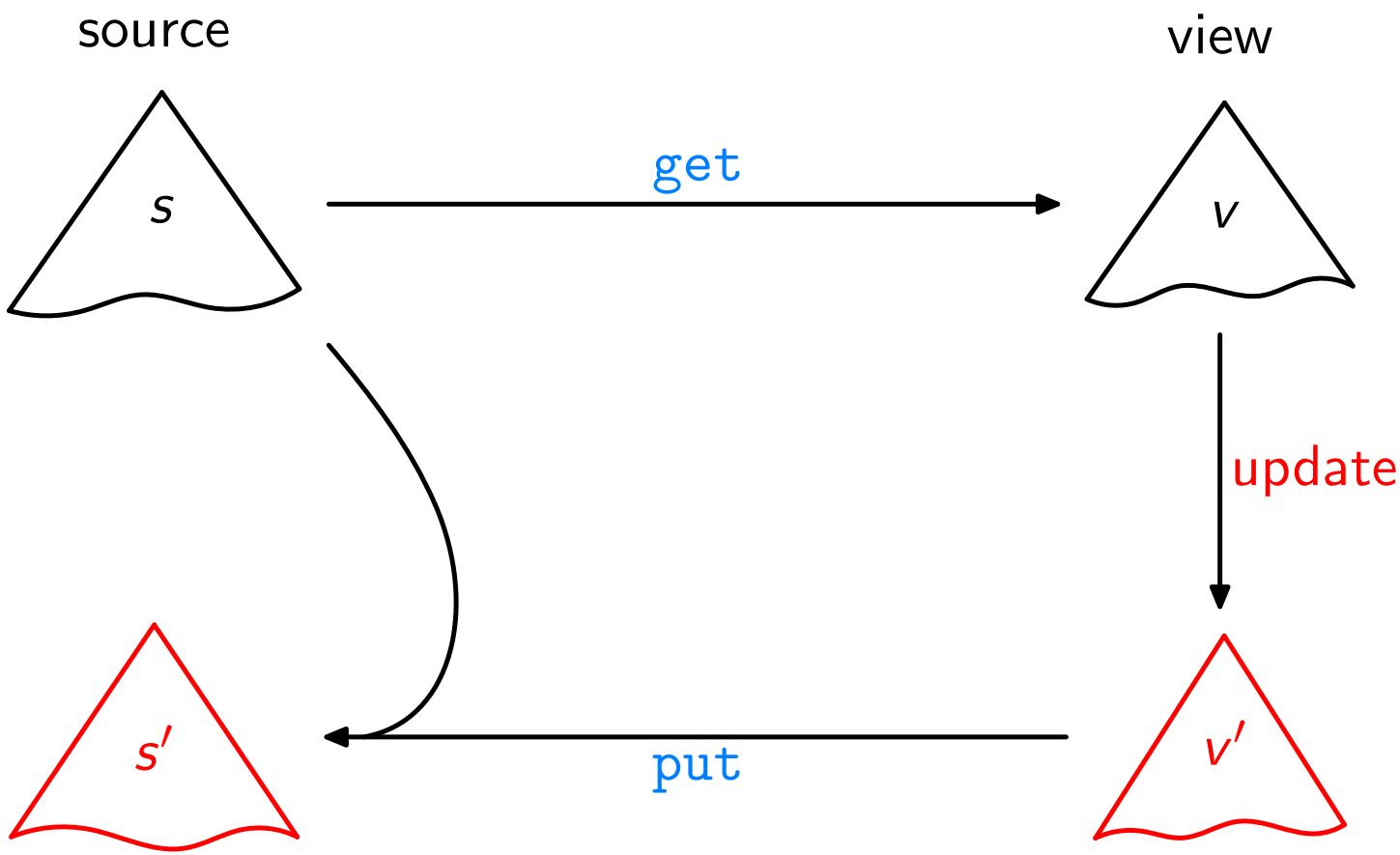

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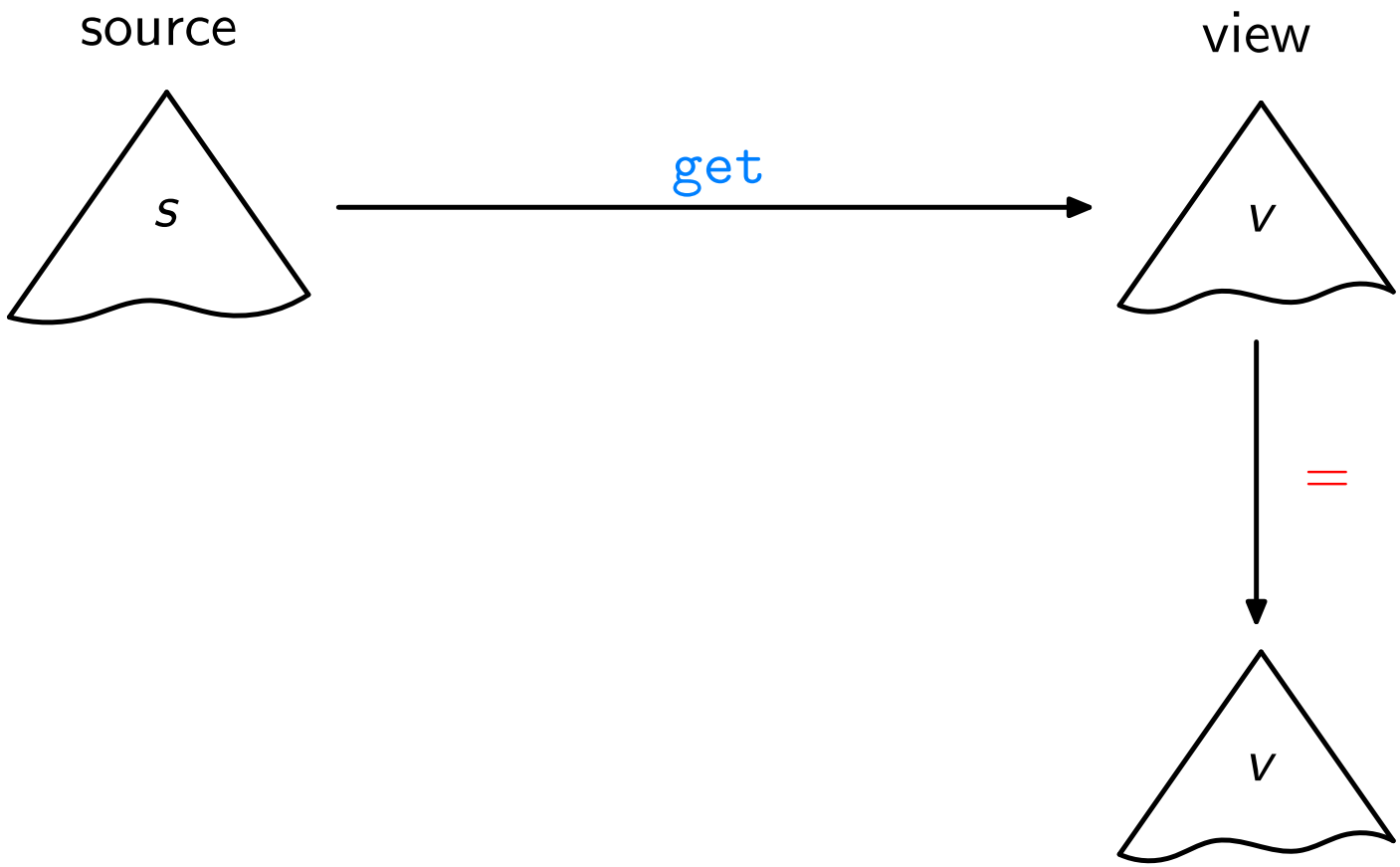

Acceptability / GetPut

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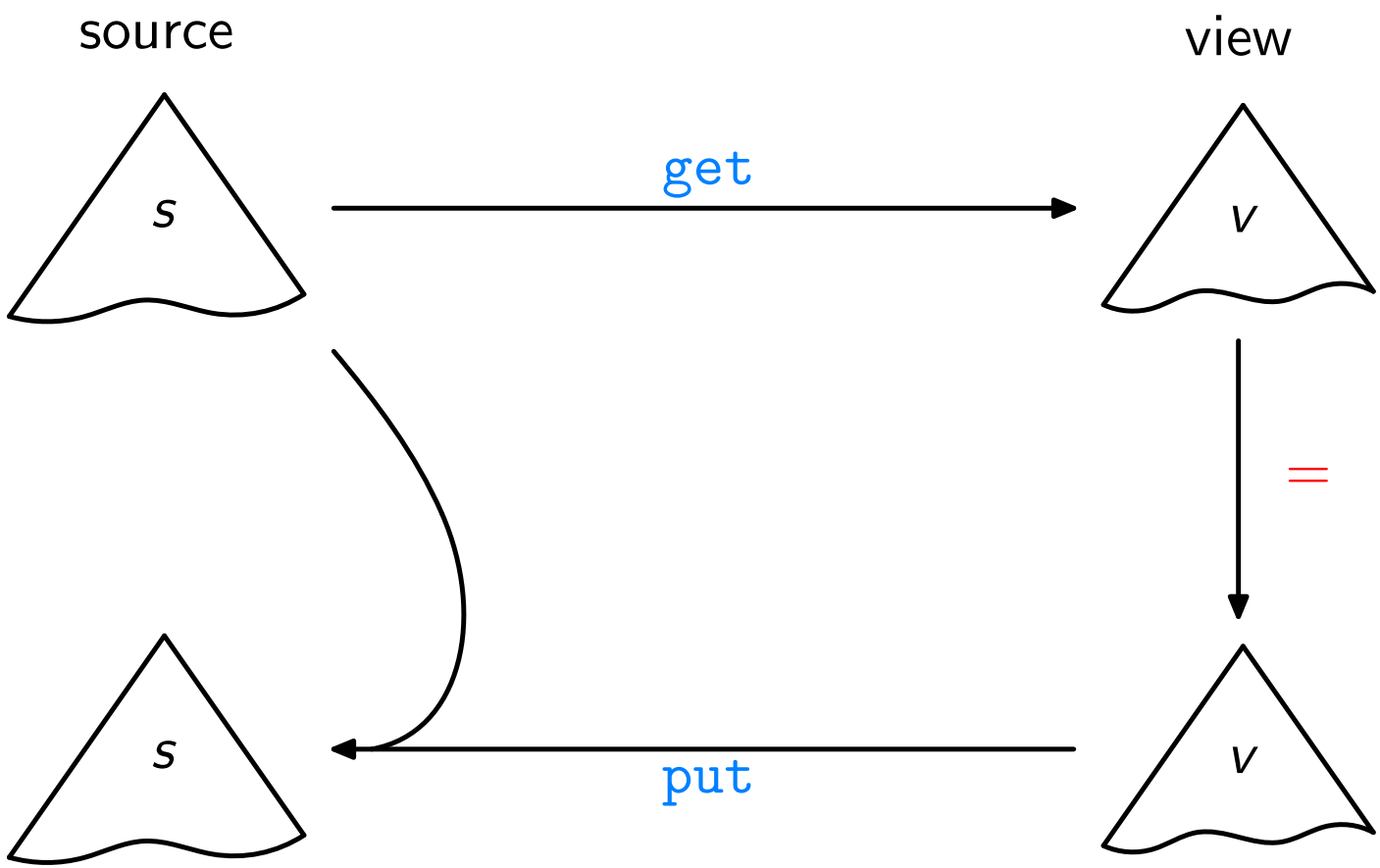

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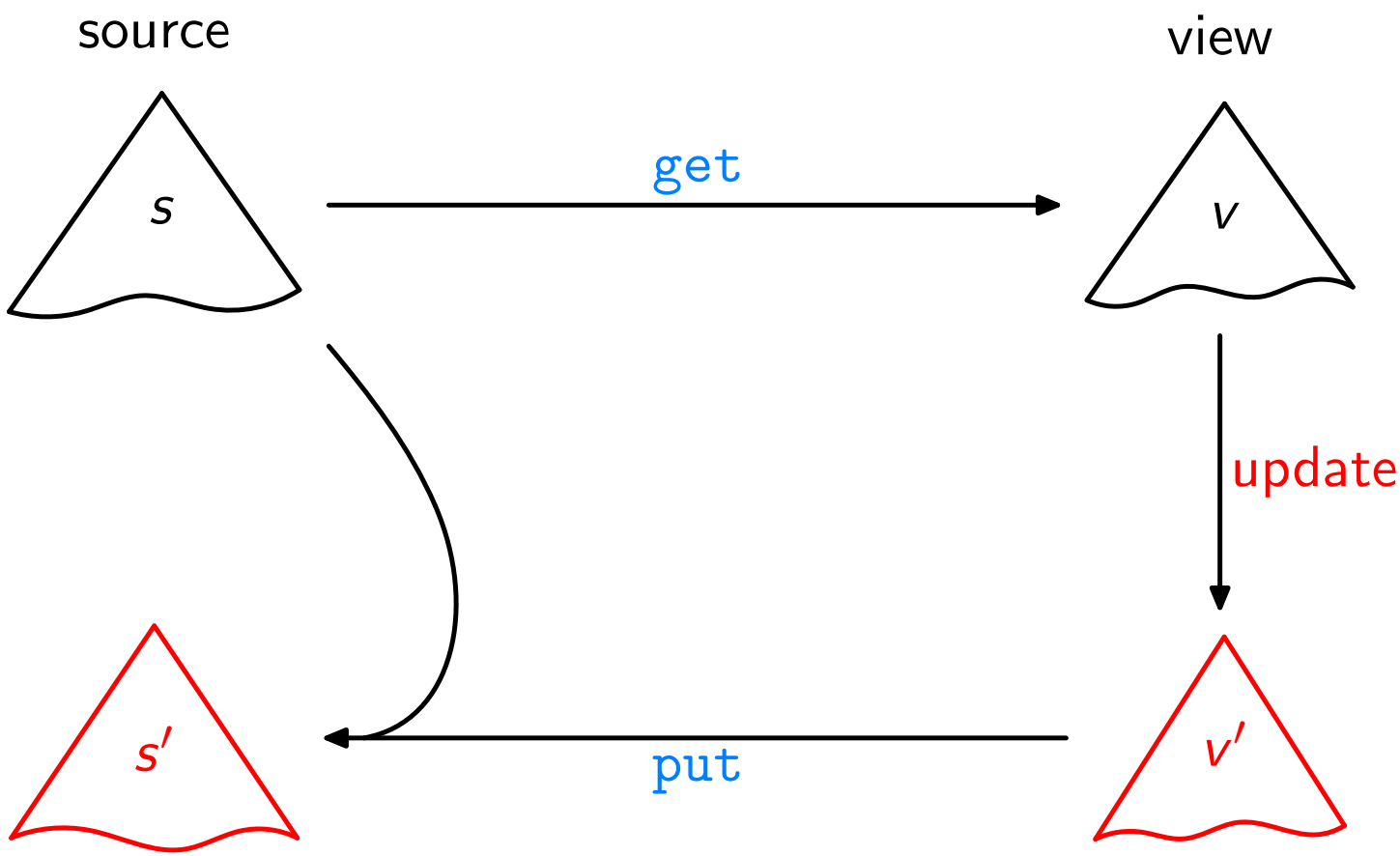

Consistency / PutGet

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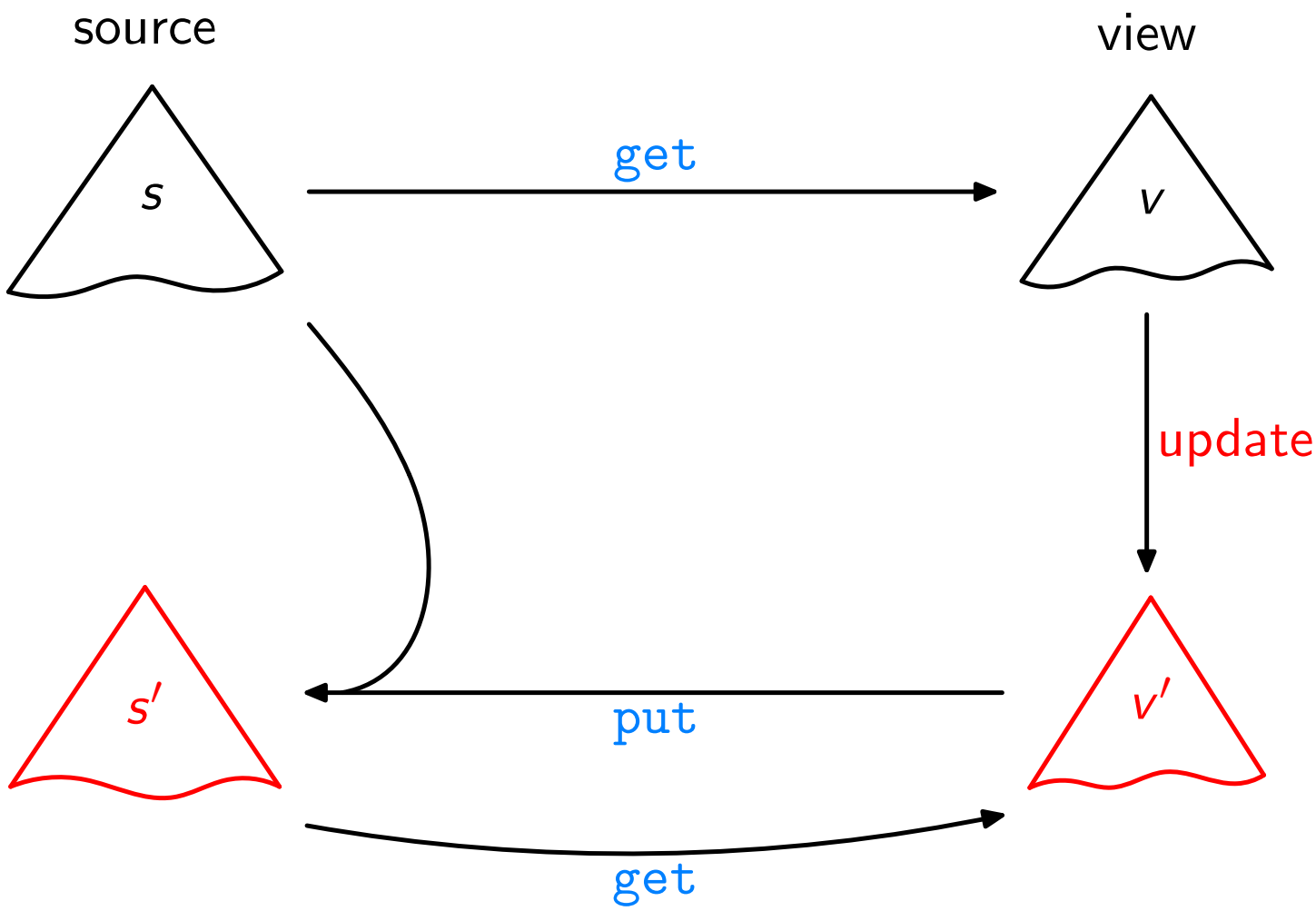

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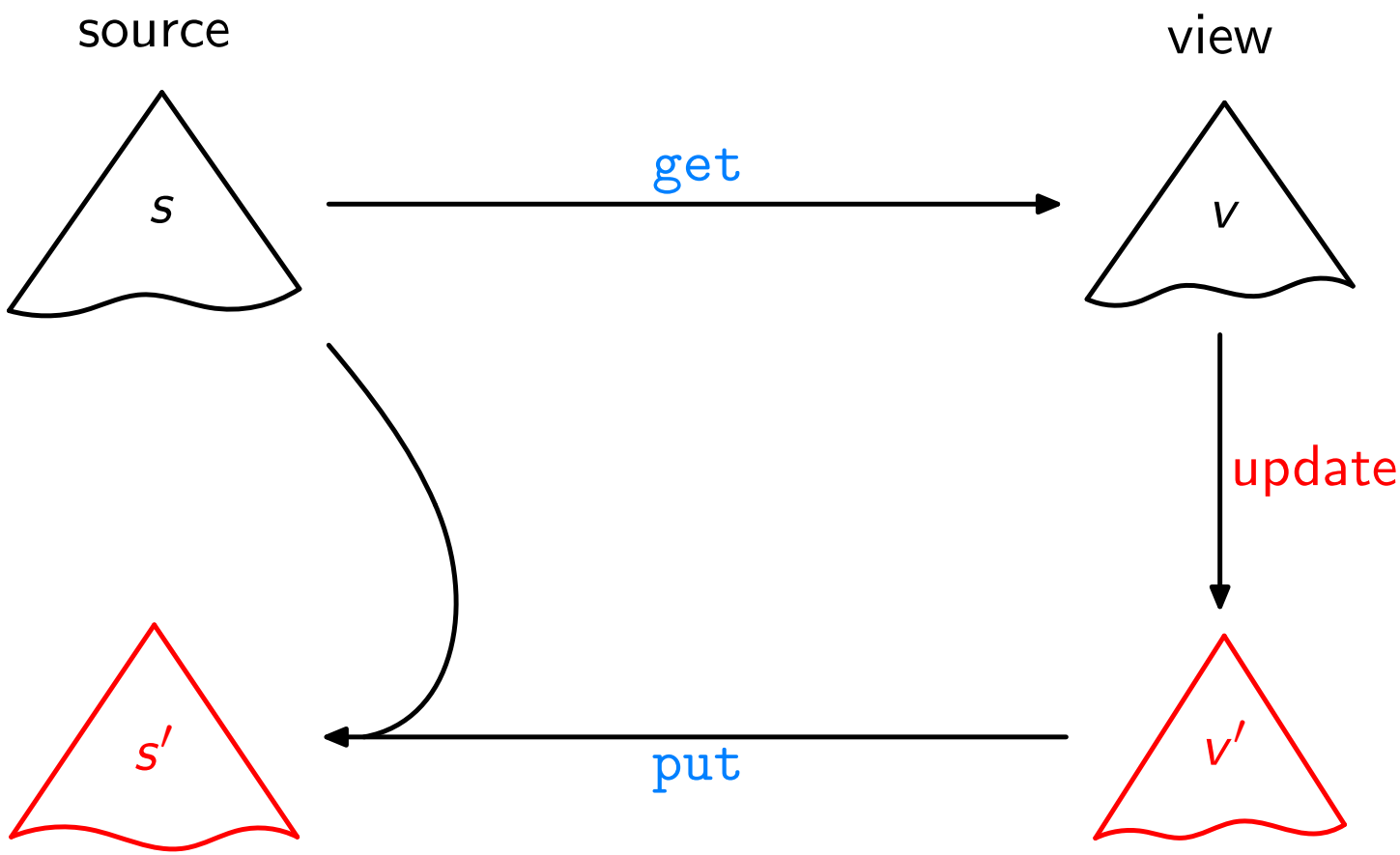

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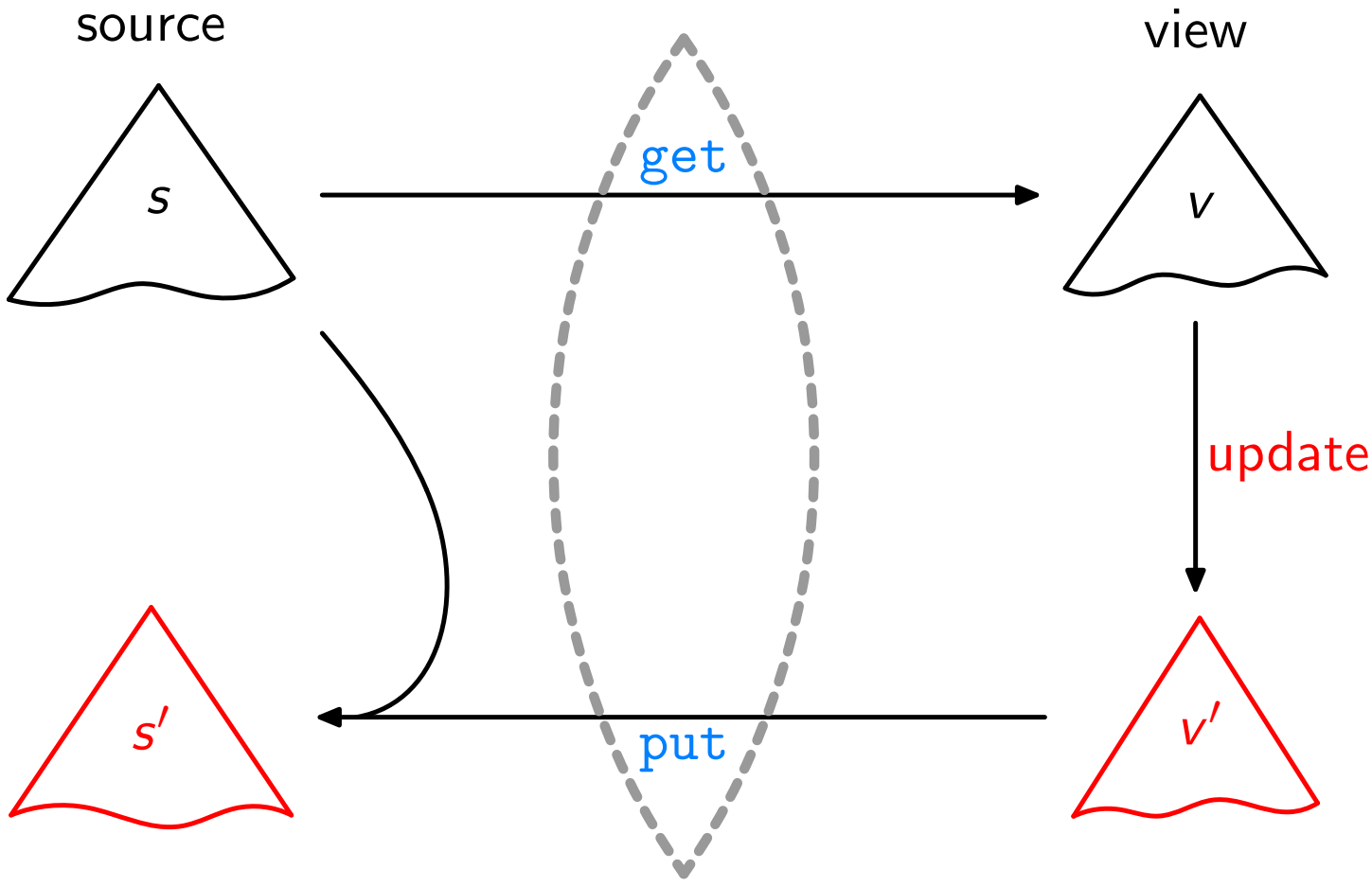

Lenses, DSLs

[Foster et al., ACM TOPLAS'07, . . .]

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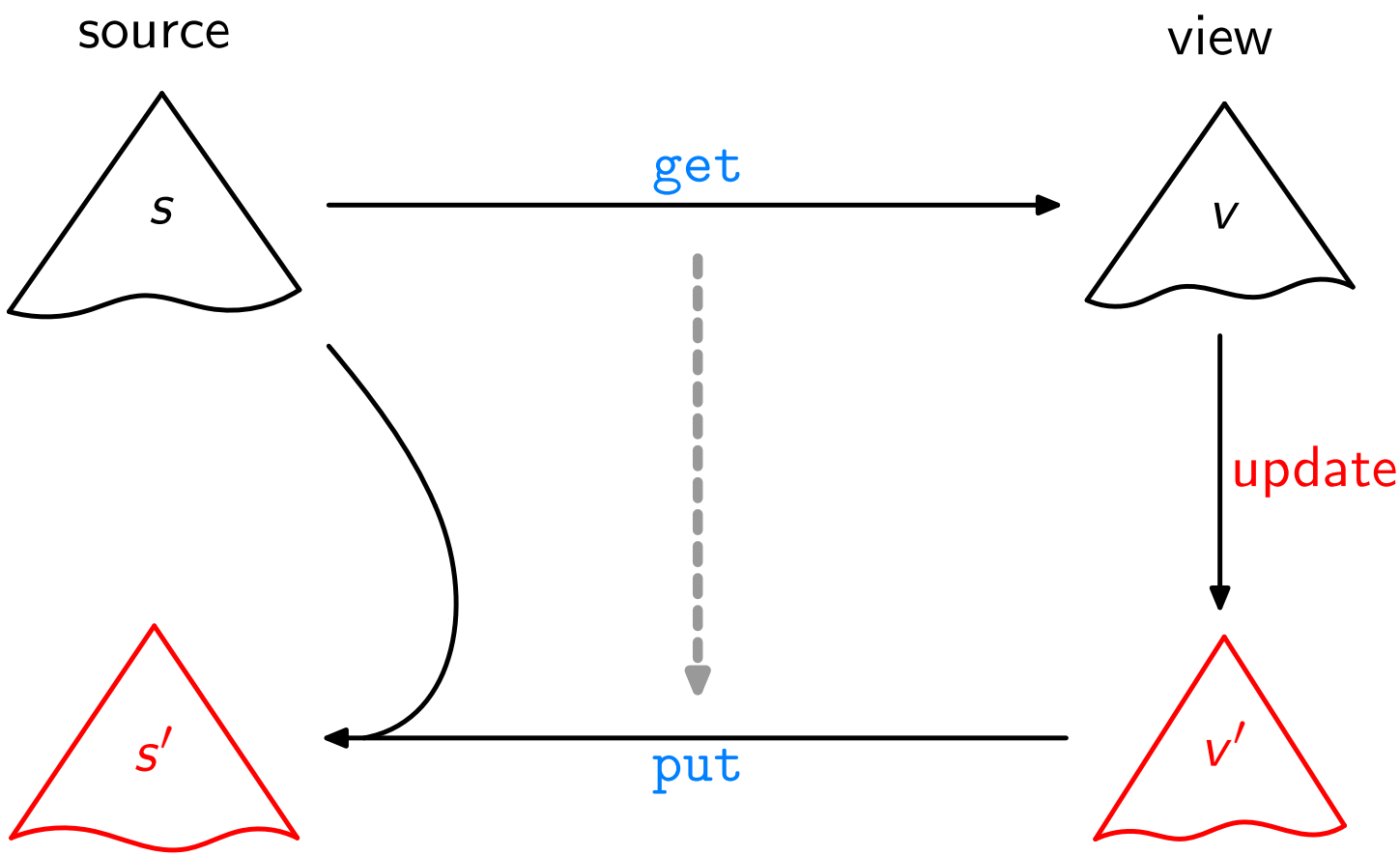

Bidirectionalization

[Matsuda et al., ICFP'07]

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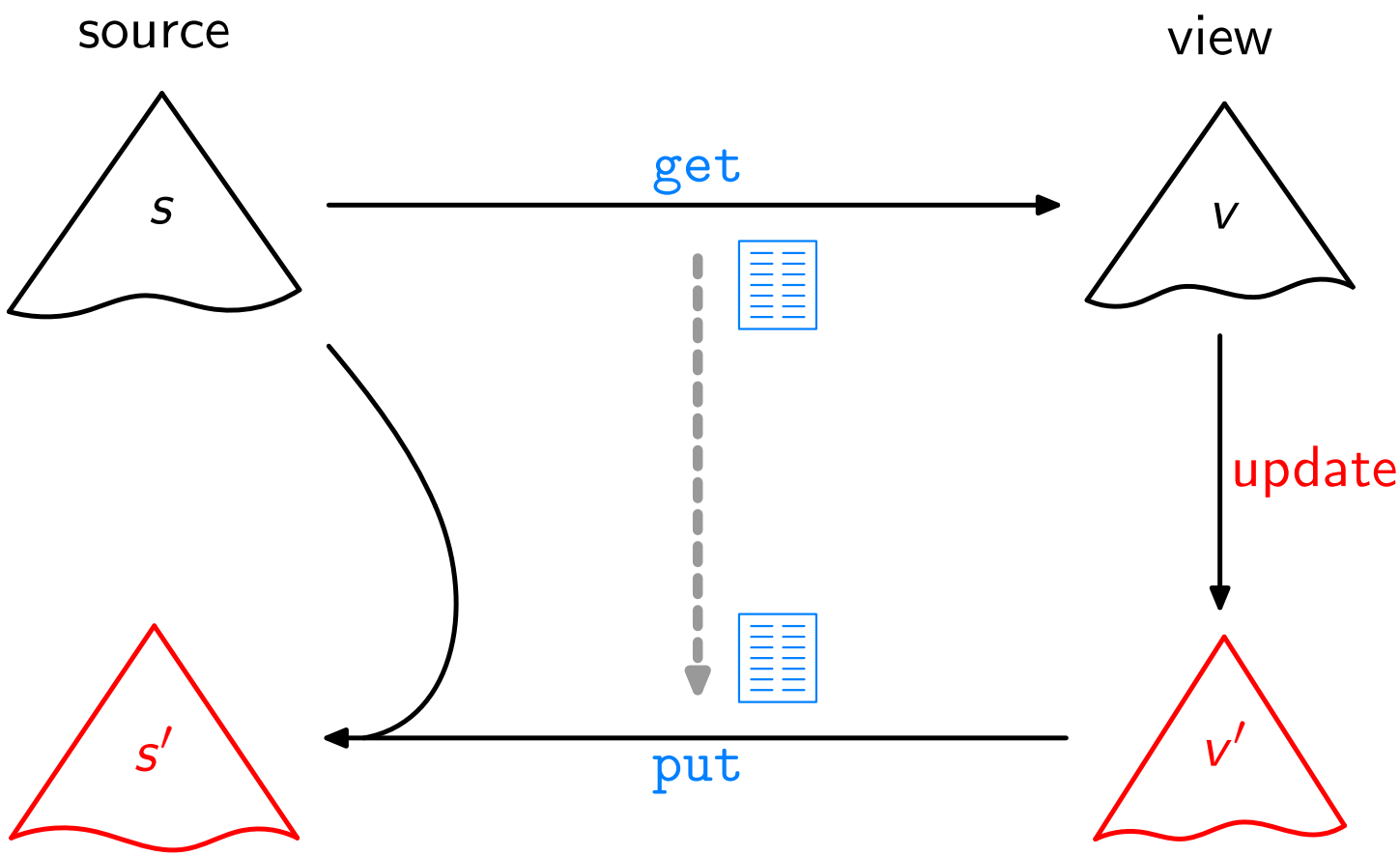
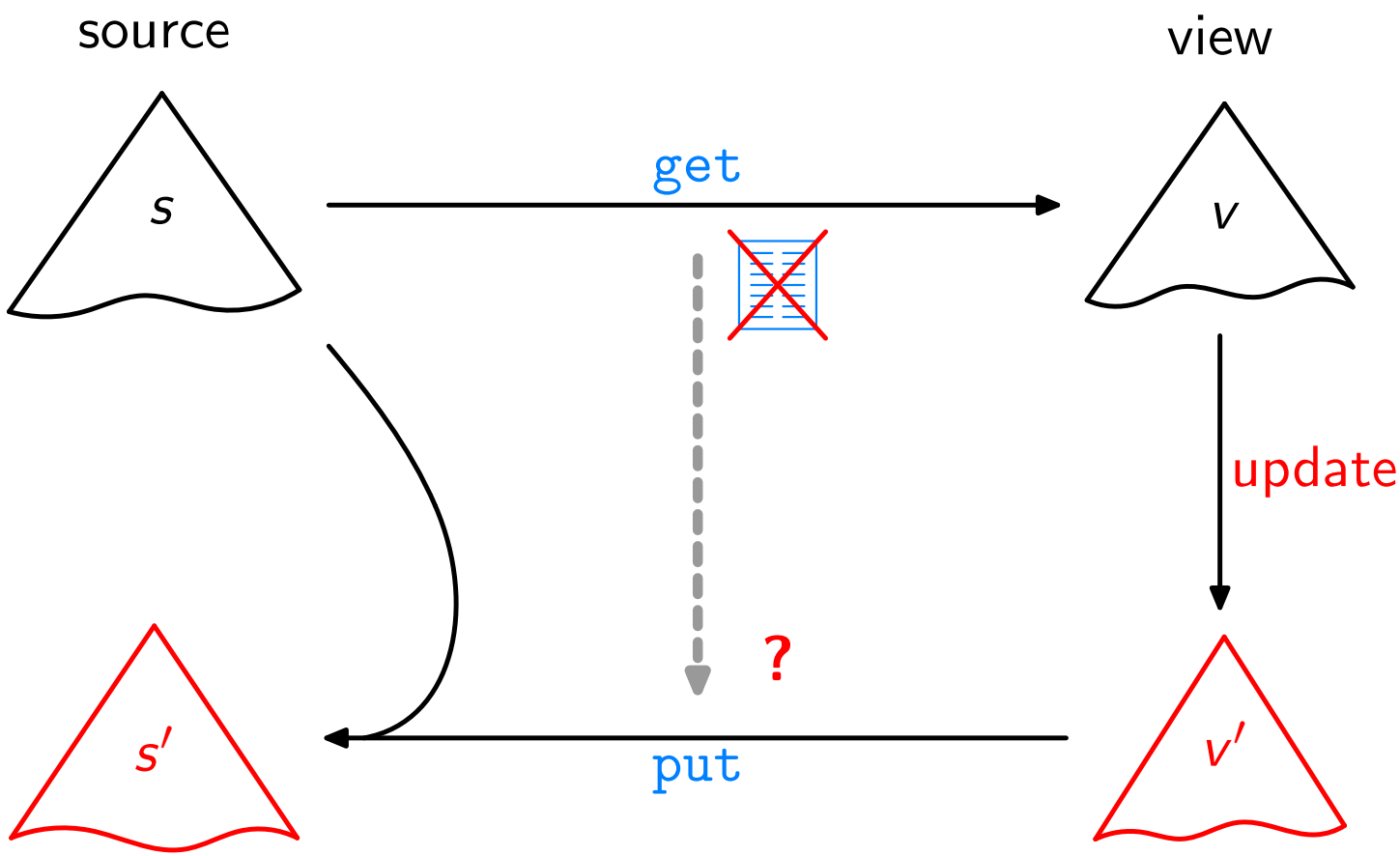


Syntactic Bidirectionalization

[Matsuda et al., ICFP'07]

Bidirectional Transformation



Semantic Bidirectionalization

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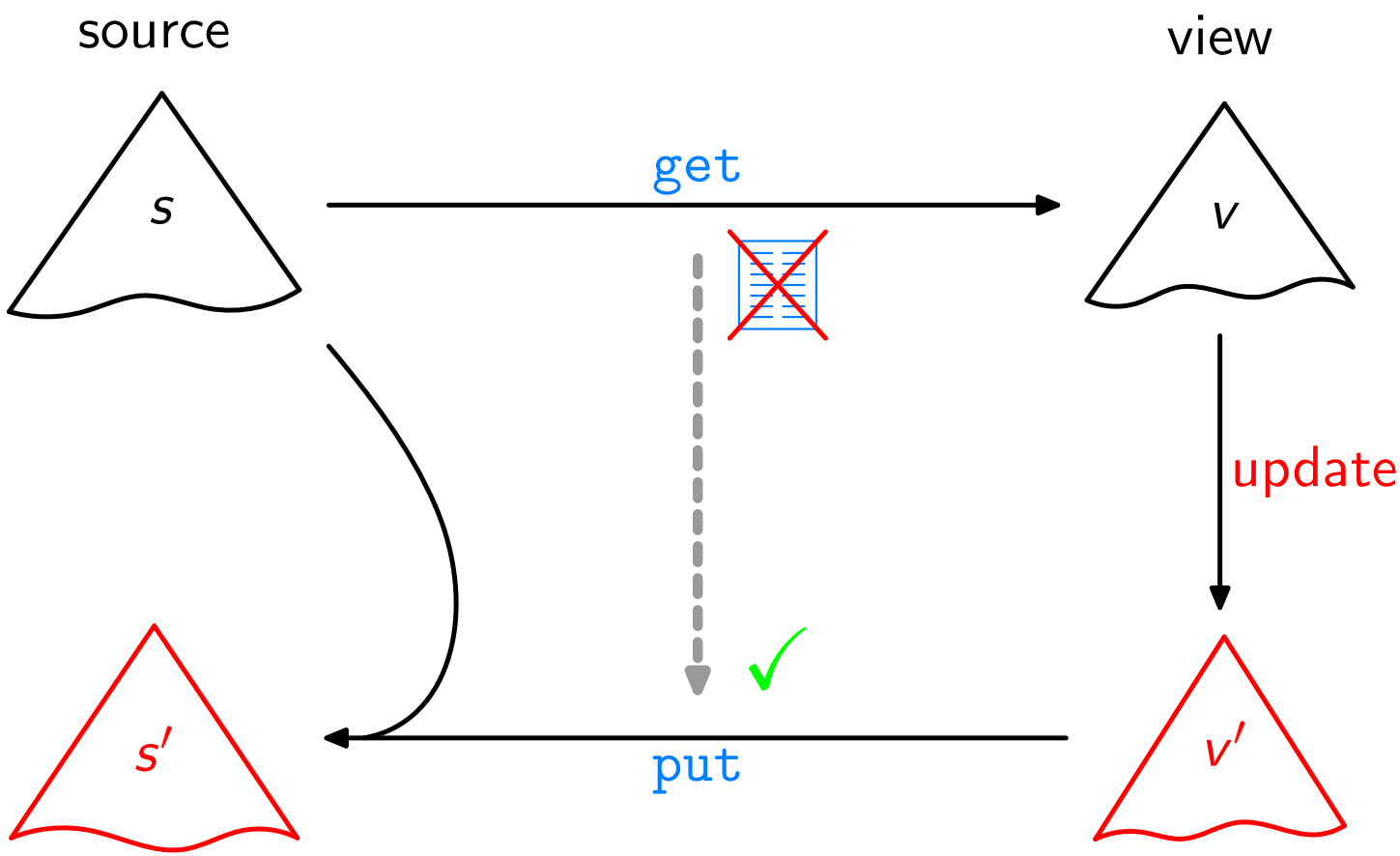

Semantic Bidirectionalization

[V., POPL'09]

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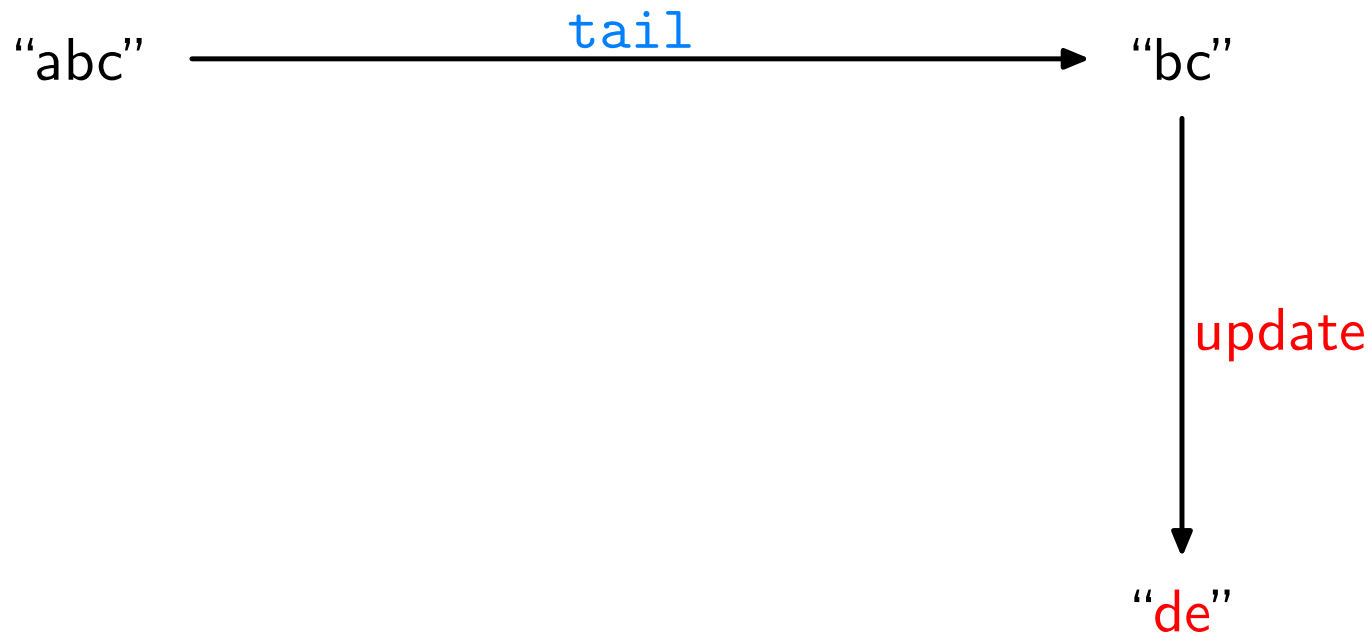

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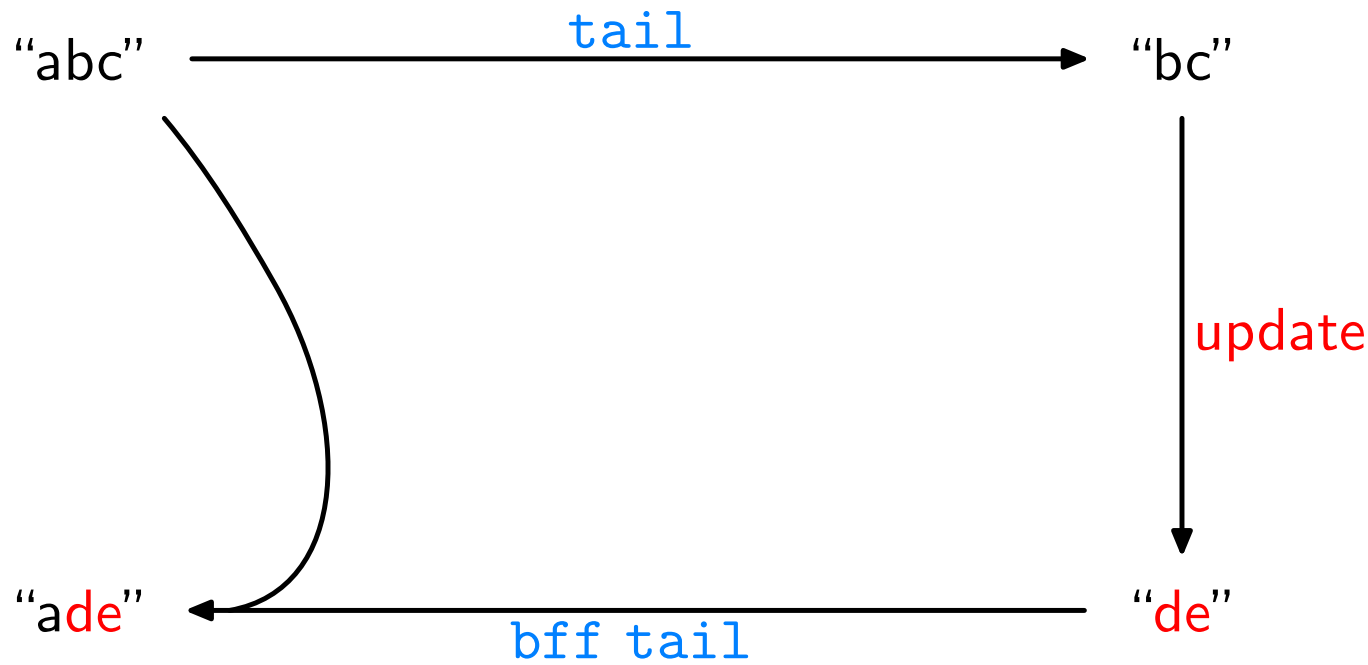

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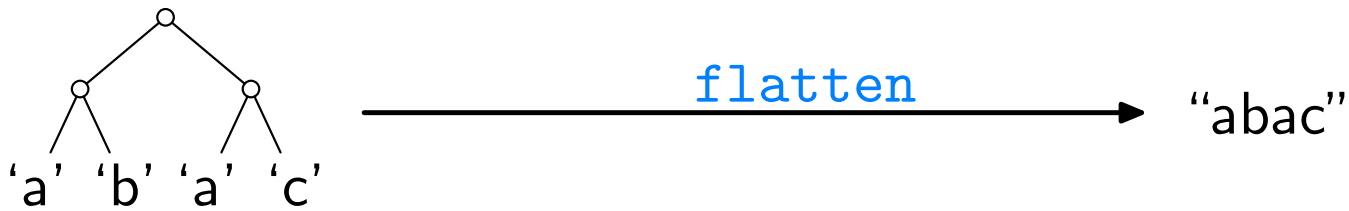

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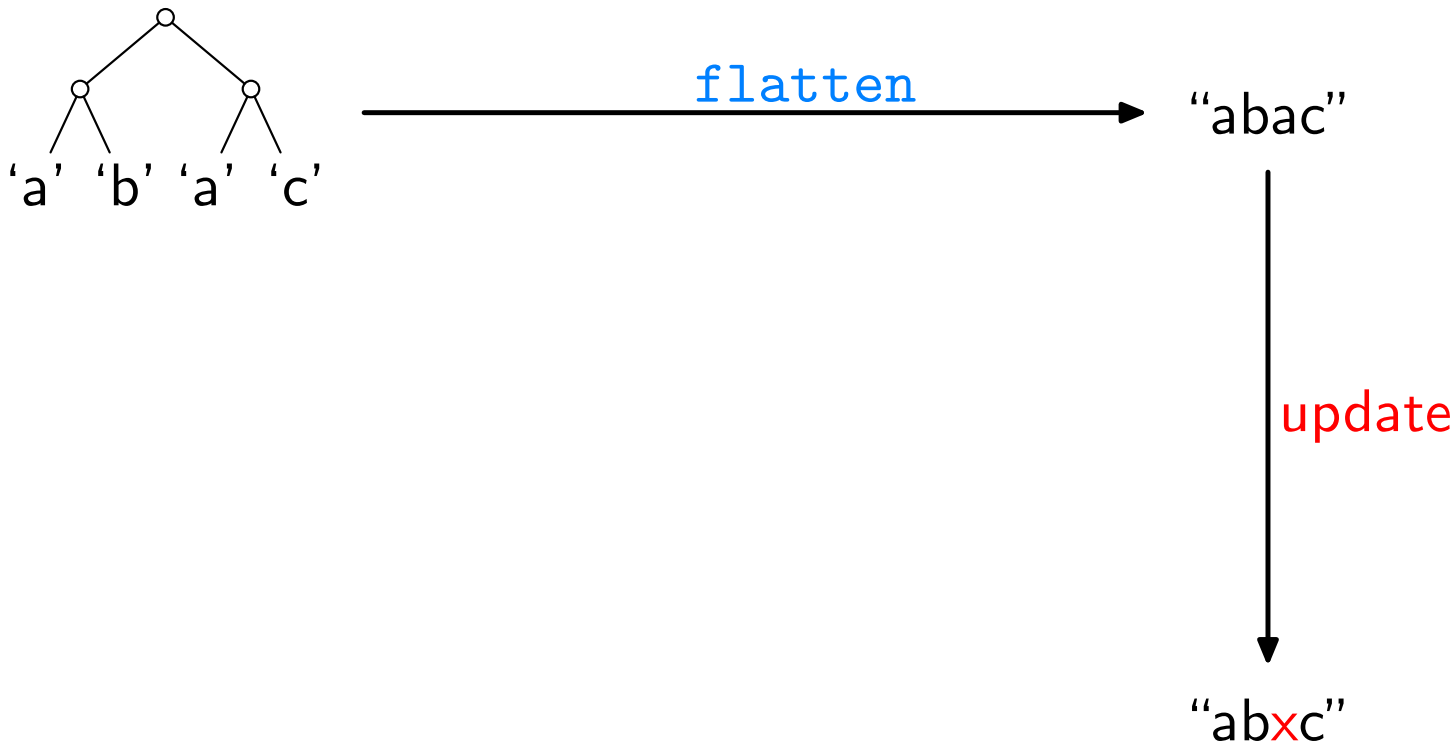

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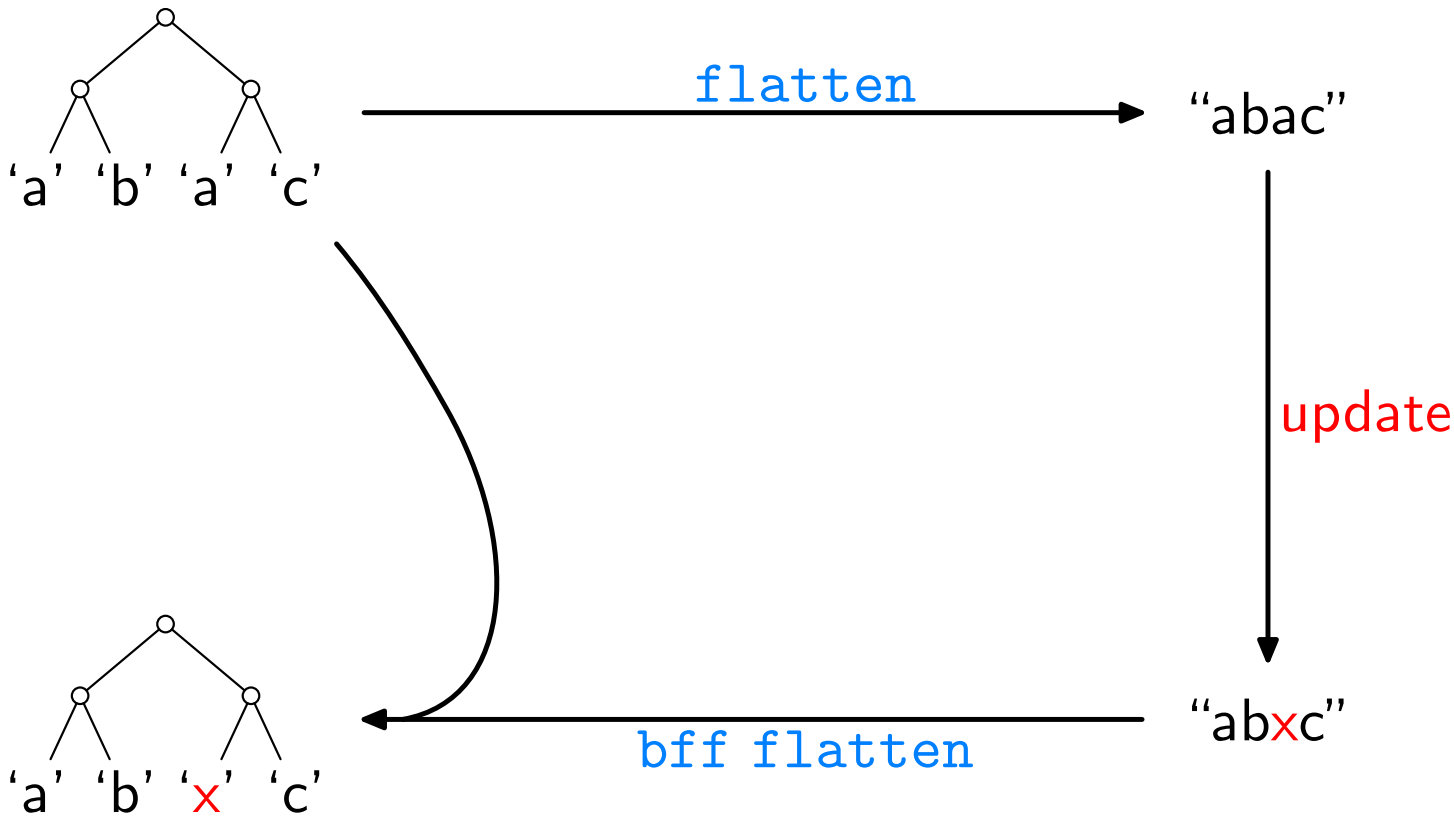

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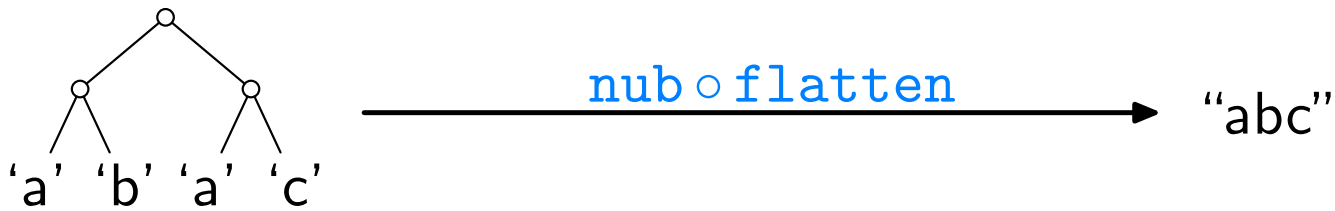

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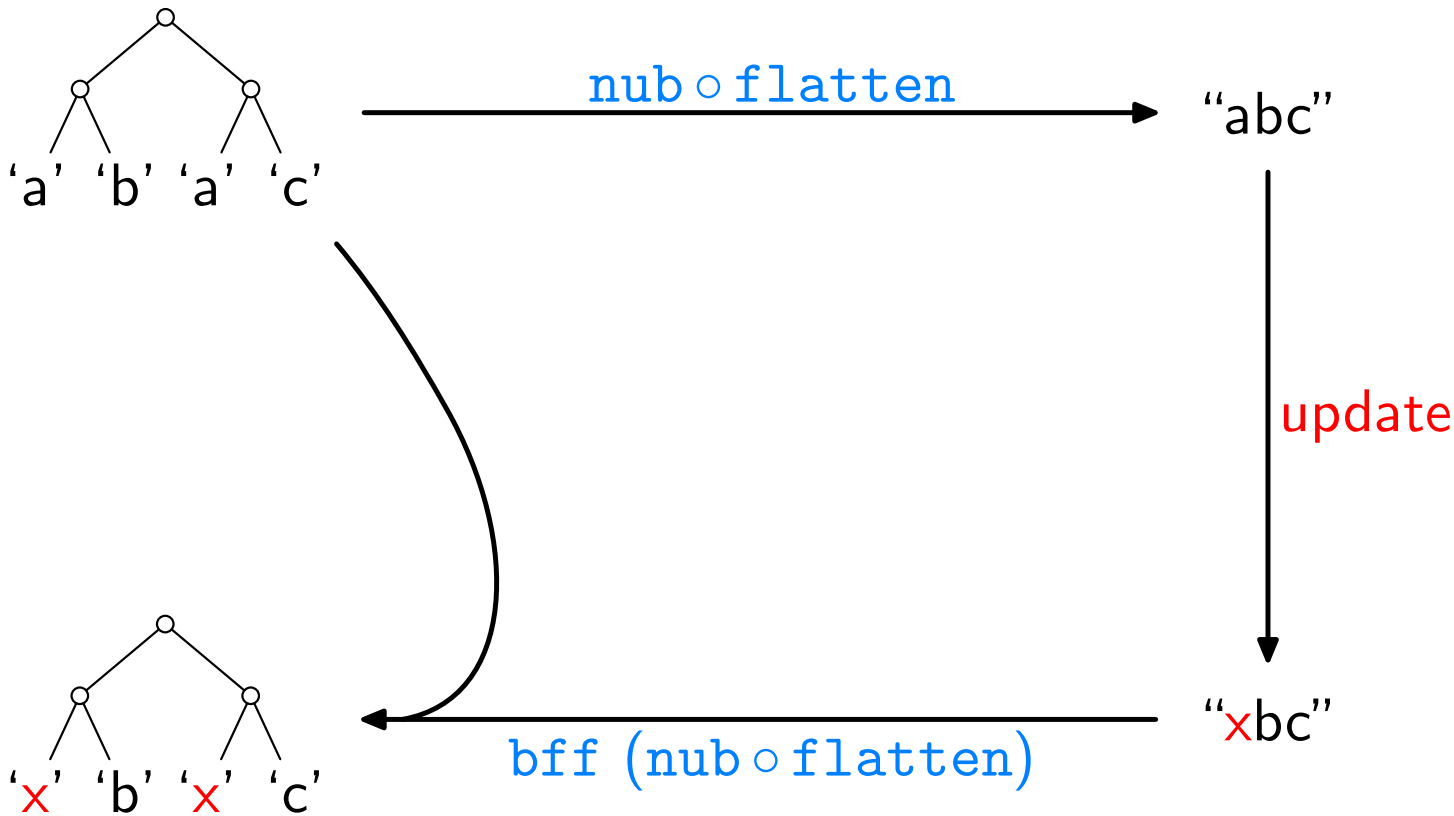

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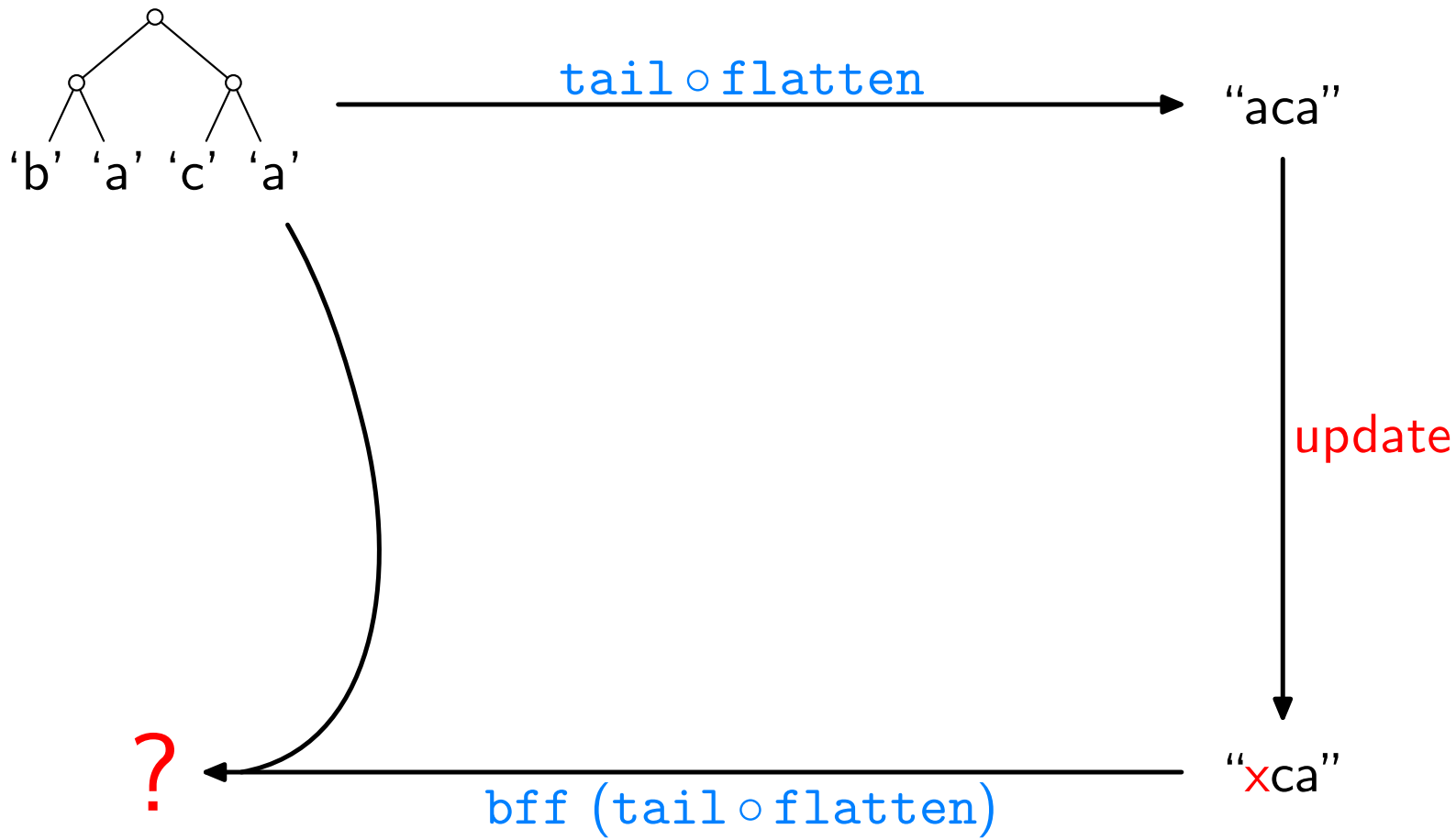

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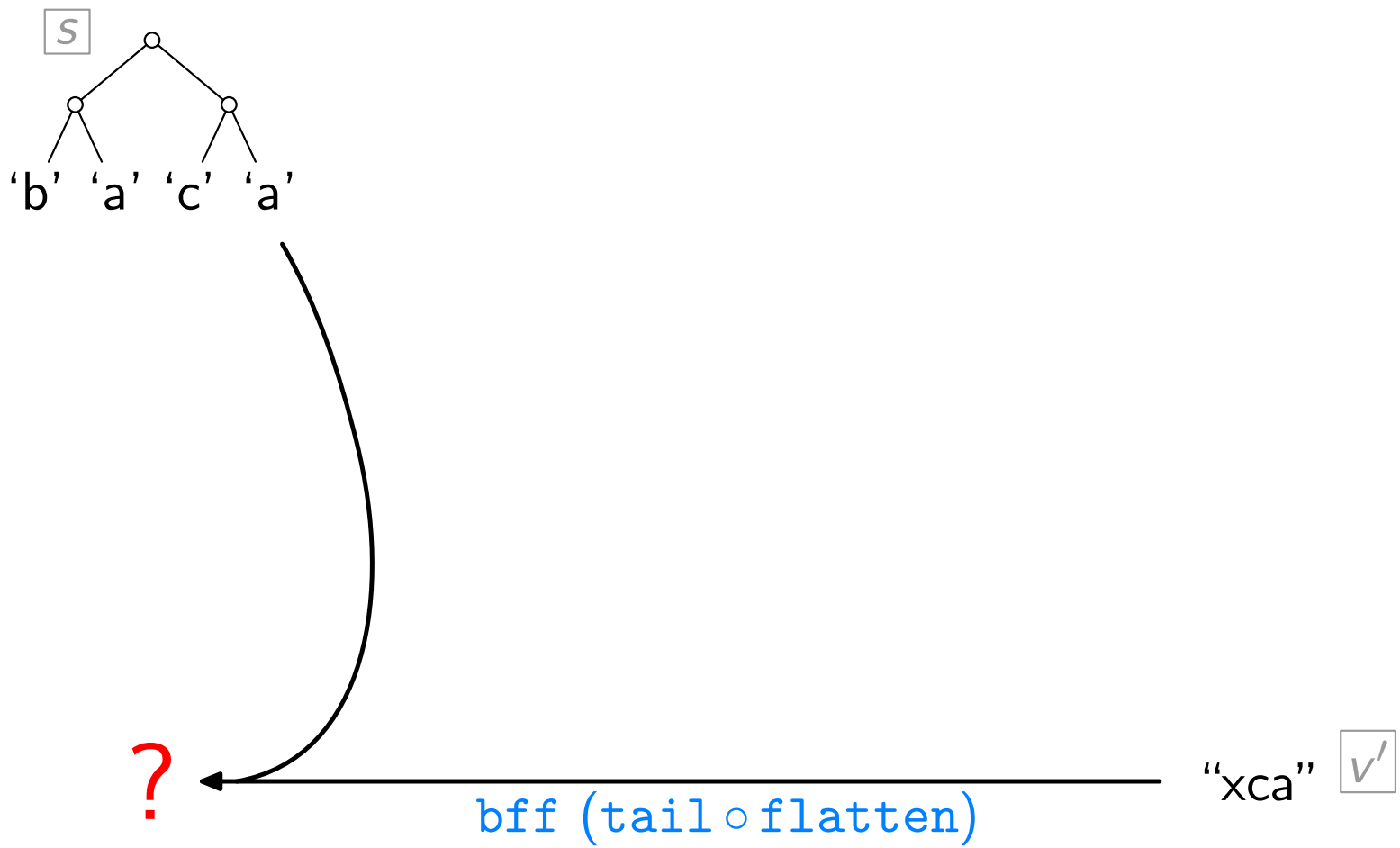

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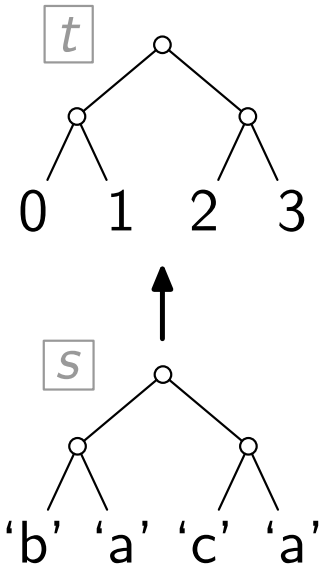

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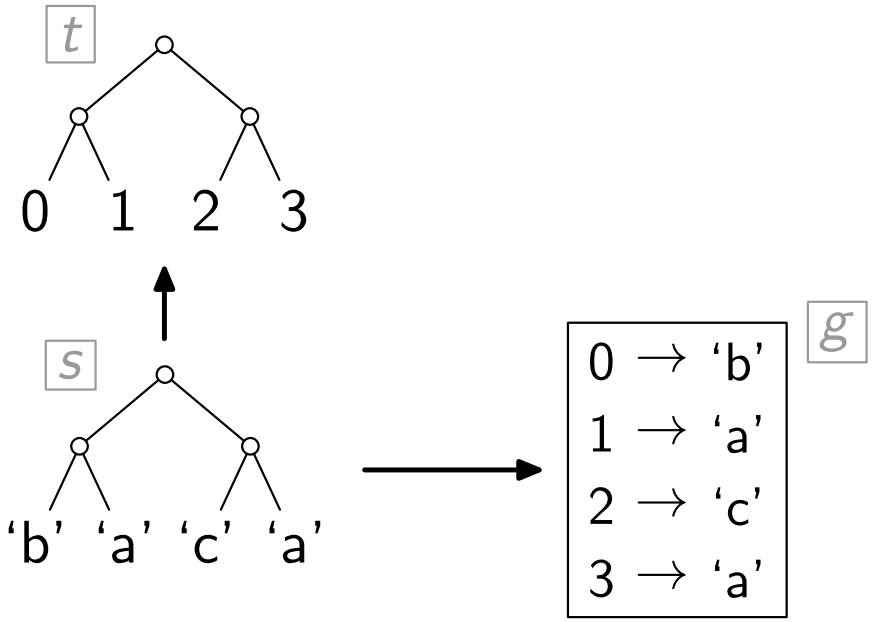

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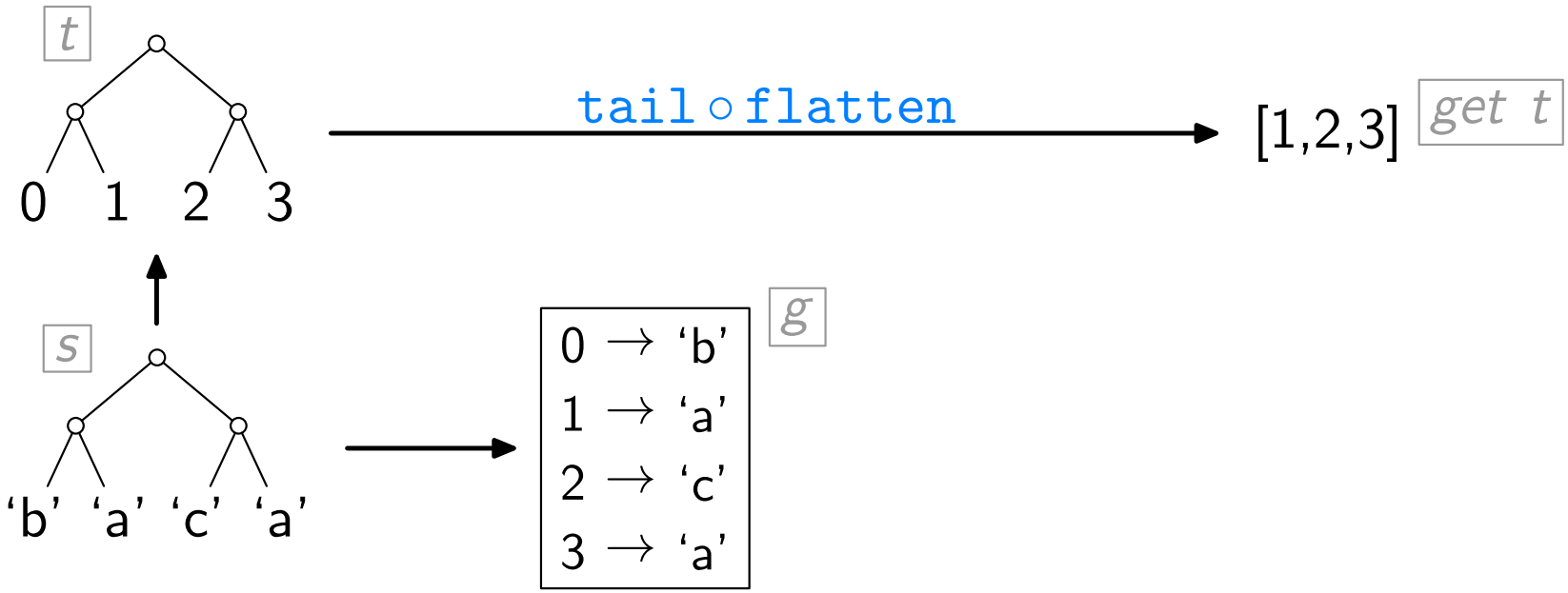

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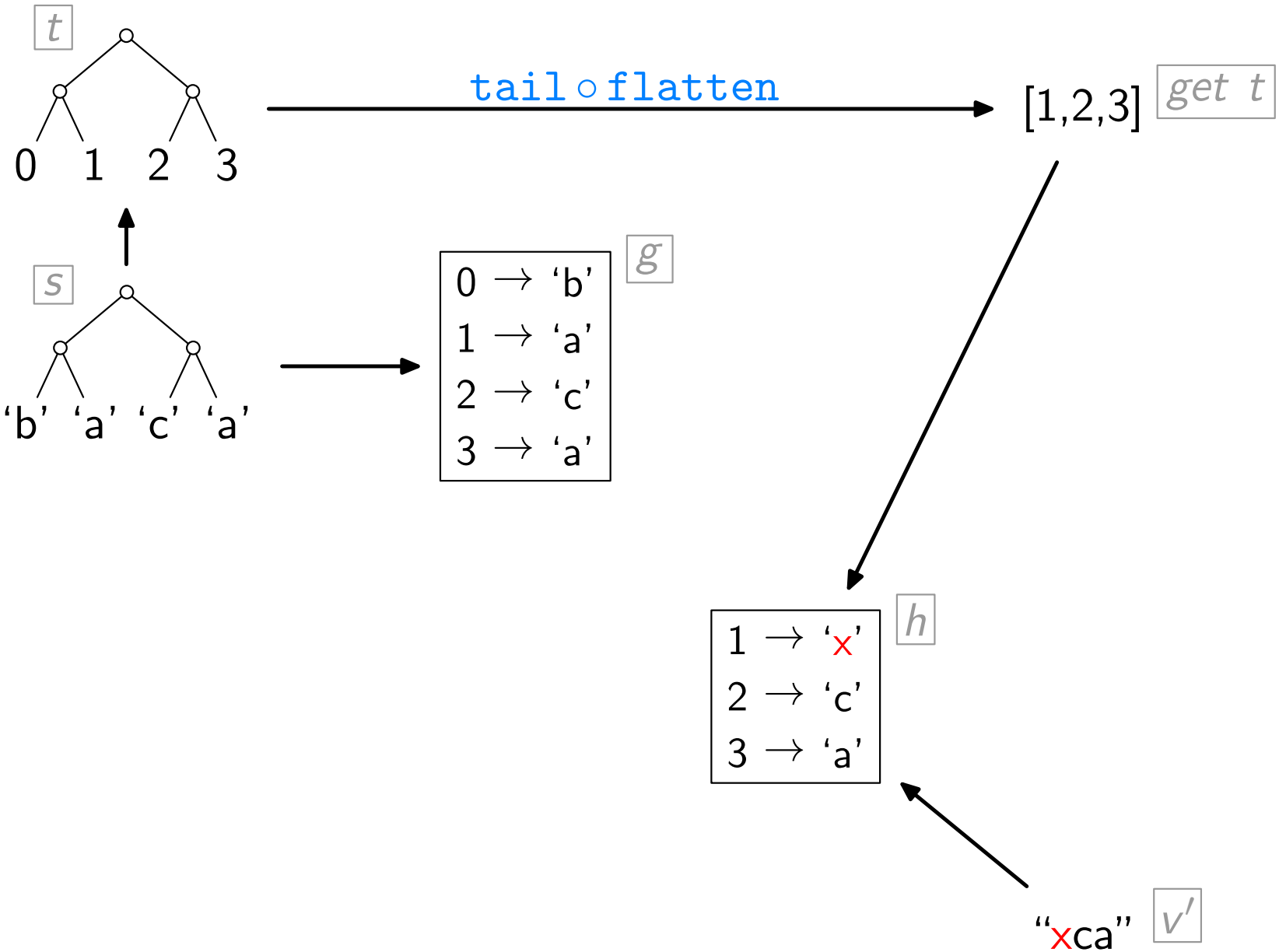

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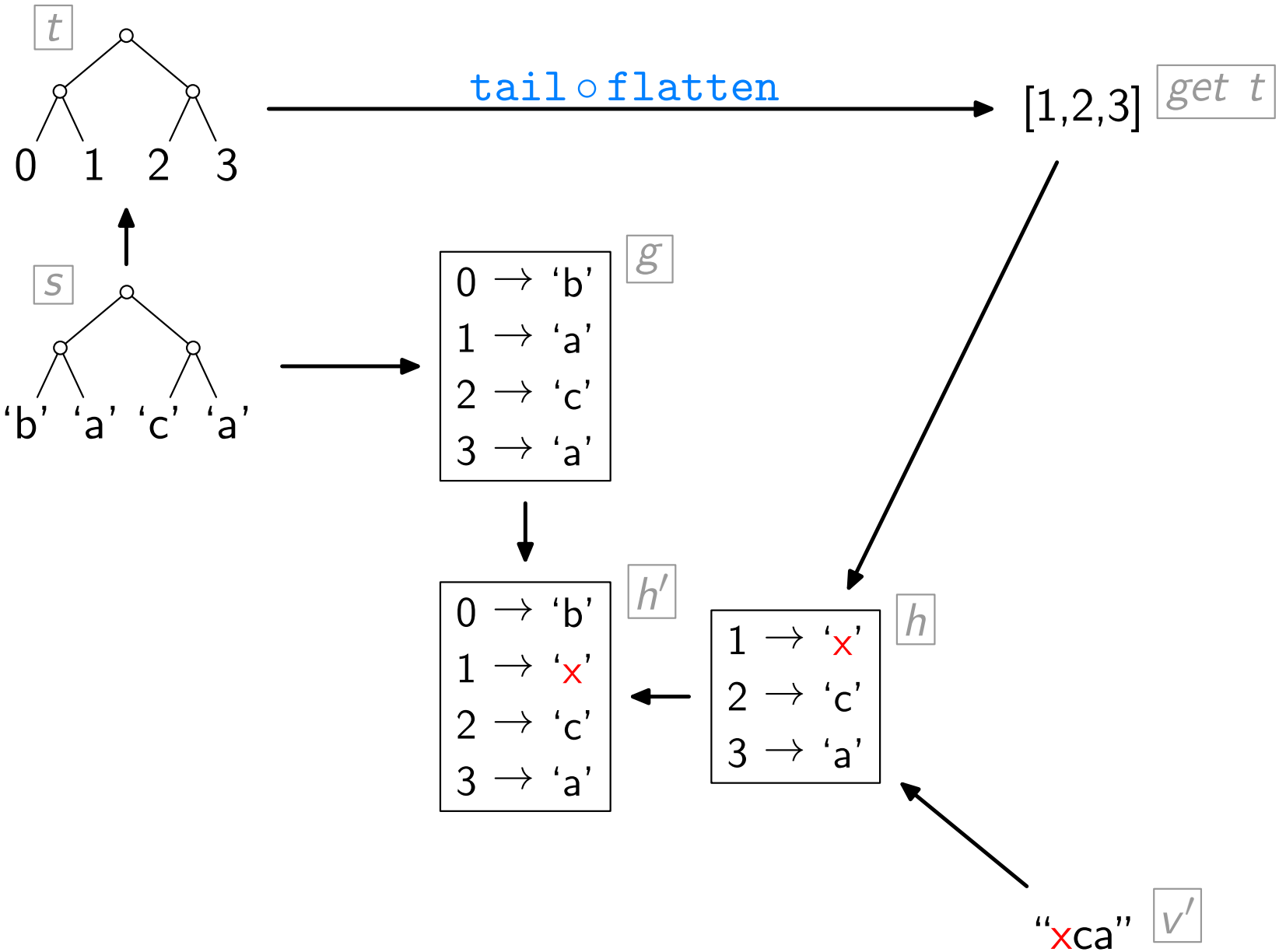

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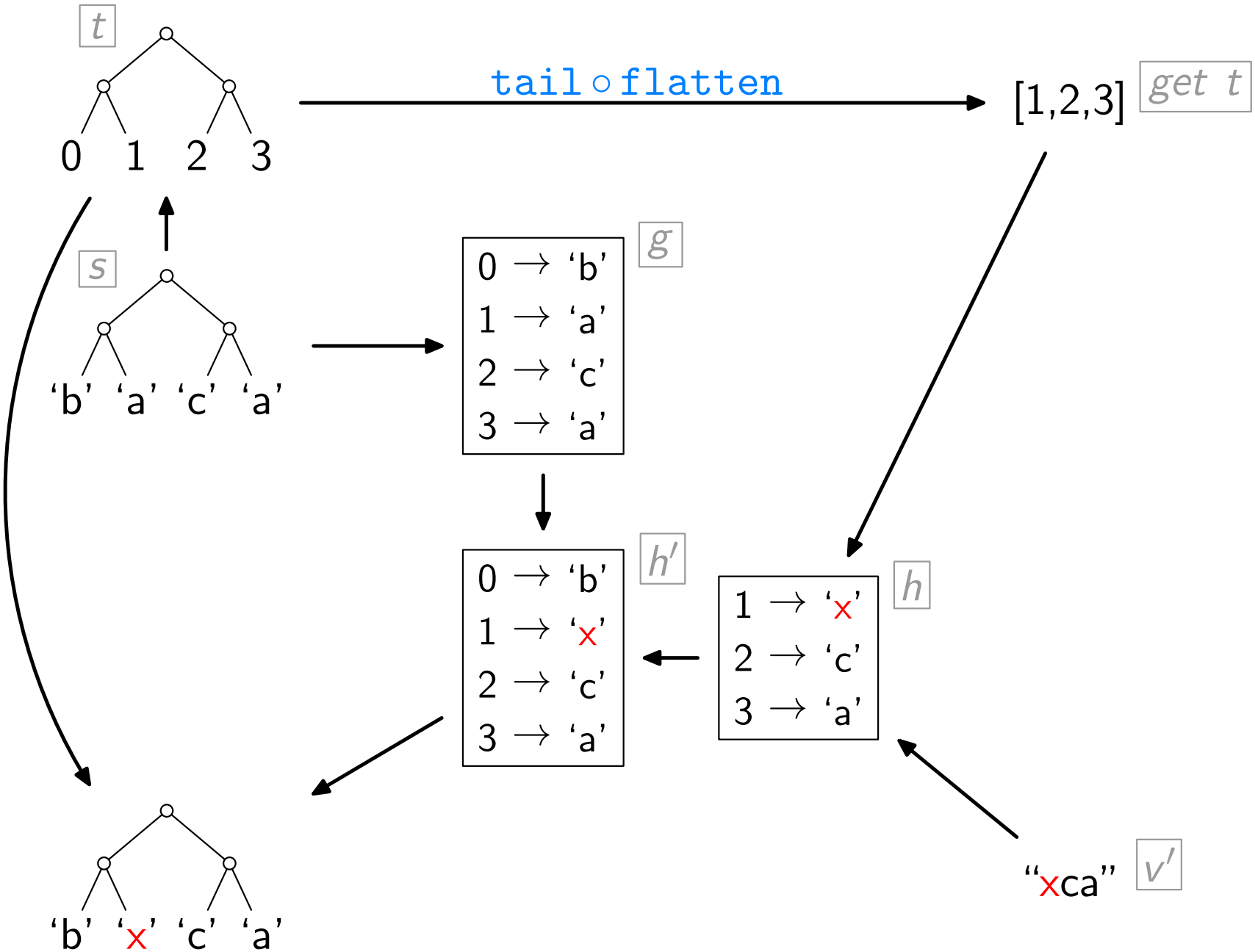

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Important: `compl` should "collapse" as much as possible.

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For a very simple setting,

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For the moment, be maximally conservative.

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\begin{aligned}
&\texttt{compl} :: [\alpha] \to (\mathsf{Int}, [\alpha]) \\
&\texttt{compl}\ s = \textbf{let}\ n = (\texttt{length}\ s) - 1 \\
&\qquad\qquad\quad t = [0..n] \\
&\qquad\qquad\quad g = \texttt{zip}\ t\ s \\
&\qquad\qquad\quad g' = \texttt{filter}\ (\lambda(i, \_) \to \texttt{notElem}\ i\ (\texttt{get}\ t))\ g \\
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\texttt{get} = \texttt{reverse} \quad &\rightsquigarrow \quad \texttt{compl}\ \text{“abcde”} = (5, [\,])
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An Inverse of $\lambda s \to (\texttt{get}\ s, \texttt{compl}\ s)$

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\begin{aligned}
&\texttt{inv} :: ([\alpha], (\mathsf{Int}, [\alpha])) \to [\alpha] \\
&\texttt{inv}\ ([\,], (0, \_)) = [\,] \\
&\texttt{inv}\ (v', (n+1, as)) = \\
&\quad \textbf{let}\ t \ = [0..n] \\
&\qquad\quad h \ = \texttt{assoc}\ (\texttt{get}\ t)\ v' \\
&\qquad\quad g' = \texttt{zip}\ (\texttt{filter}\ (\lambda i \to \texttt{notElem}\ i\ (\texttt{get}\ t))\ t)\ as \\
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inv :: ([α], (Int, [α])) → [α]
inv ([], (0, _)) = []
inv (v', (n + 1, as)) =
  let t  = [0..n]
      h  = assoc† (get t) v'
      g' = zip (filter (λi → notElem i (get t)) t) as
      h' = h ++ g'
  in  map (λi → fromJust (lookup i h')) t
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† Can be thought of as `zip` for the moment.

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\texttt{get} = \texttt{take}\ 3 \quad &\rightsquigarrow \quad \texttt{inv}\ (\text{“xyz”}, (5, [\text{'d'}, \text{'e'}])) = \text{“xyzde”}
\end{aligned}
$$

† Can be thought of as `zip` for the moment.

Correctness

To prove formally:

- $\texttt{inv}\ (\texttt{get}\ s, \texttt{compl}\ s) = s$
- if $\texttt{inv}\ (v, c)$ defined, then $\texttt{get}\ (\texttt{inv}\ (v, c)) = v$
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Use a free theorem [Wadler, FPCA'89], namely that for every

$$\texttt{get} :: [\alpha] \to [\alpha]$$

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Given an arbitrary list s of length $n+1$, set $l = [0..n]$, $f = (s\,!!)$, leading to:

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Altogether, So Far:

```
compl :: [α] → (Int, [α])
compl s = let n  = (length s) − 1
              t  = [0..n]
              g  = zip t s
              g' = filter (λ(i, _) → notElem i (get t)) g
          in  (n + 1, map snd g')

inv :: ([α], (Int, [α])) → [α]
inv ([], (0, _)) = []
inv (v', (n + 1, as)) =
  let t  = [0..n]
      h  = assoc (get t) v'
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      h' = h ++ g'
  in  map (λi → fromJust (lookup i h')) t
```

“Fusion”

Inlining `compl` and `inv` into `put`, plus some clever rewriting:

$$
\begin{aligned}
&\texttt{put}\ [\,]\ [\,] = [\,] \\
&\texttt{put}\ s\ v' = \textbf{let}\ n = (\texttt{length}\ s) - 1 \\
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&\qquad\qquad\quad\ h = \texttt{assoc}\ (\texttt{get}\ t)\ v' \\
&\qquad\qquad\quad\ h' = h +\!\!+ g' \\
&\qquad\qquad\ \textbf{in}\ \ \texttt{seq}\ h\ (\texttt{map}\ (\lambda i \to \texttt{fromJust}\ (\texttt{lookup}\ i\ h'))\ t)
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Inlining `compl` and `inv` into `put`, plus some clever rewriting:

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put [] [] = []
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assoc []       []       = []
assoc (i : is) (b : bs) = let m = assoc is bs
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                                Nothing          → (i, b) : m
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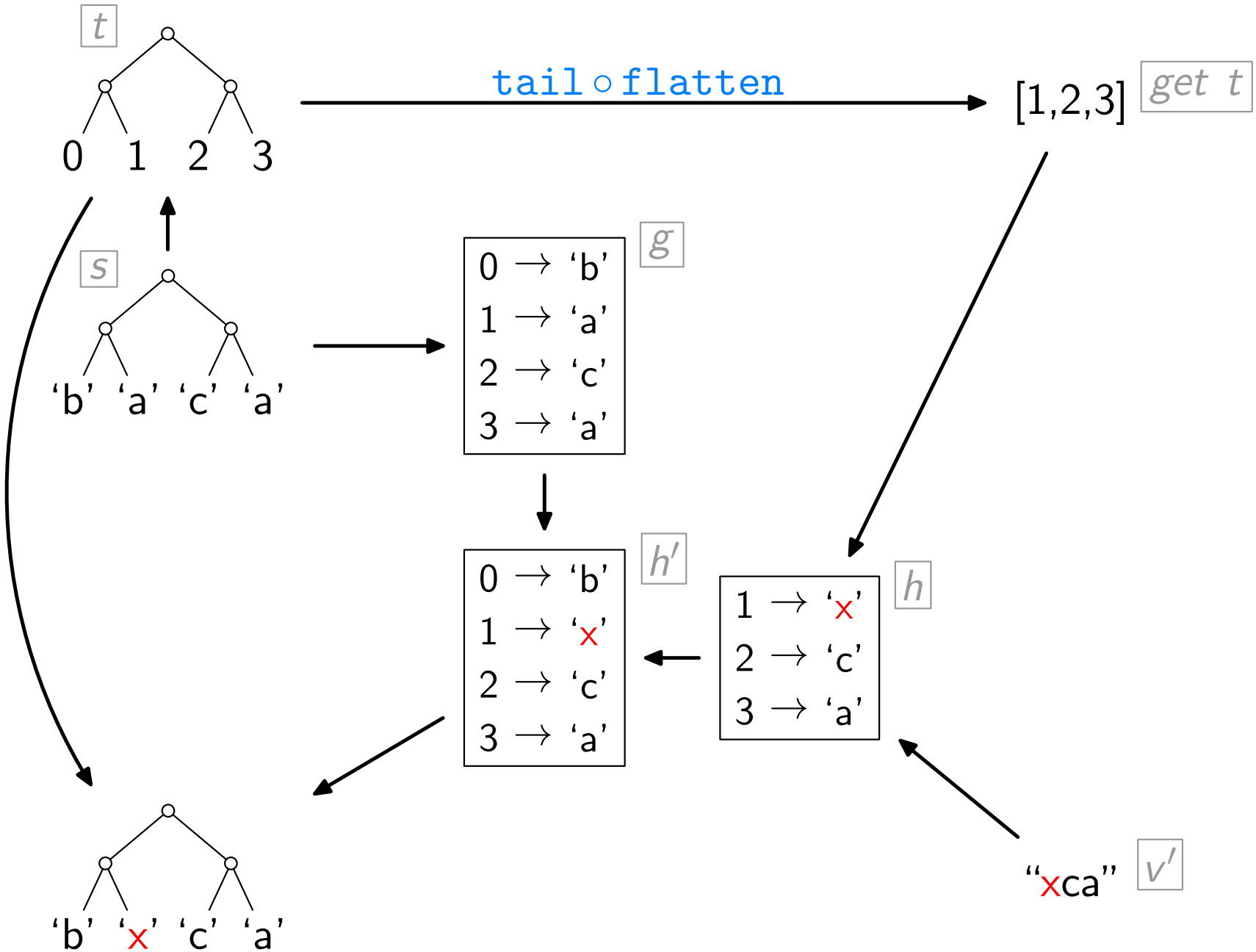
Actual code only slightly more elaborate!

Overview of the Bidirectionalization Method



Extending the Technique

Major Problem:

- Shape-affecting updates lead to failure.

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1. length of the source list
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- Being maximally conservative this way often does not “collapse enough”.
- For example:

`get = tail` $\rightsquigarrow$ `put` “abcde” “xyz” fails precisely because `compl` “abcde” $= (5, [\text{'a'}])$

Assuming Shape-Injectivity

So assume there is a function

$$\texttt{shapeInv} :: \mathsf{Int} \to \mathsf{Int}$$

with, for every source list s,

$$\texttt{length}\ s = \texttt{shapeInv}\ (\texttt{length}\ (\texttt{get}\ s))$$

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But how to obtain `shapeInv` ???

Just for experimentation:

```
shapeInv :: Int → Int
shapeInv lv = head [n + 1 | n ← [0..], (length (get [0..n])) == lv]
```

Some Tests

Works quite nicely in some cases:

$$\texttt{get} = \texttt{tail} \quad \rightsquigarrow \quad \texttt{put}\ \text{“abcde”}\ \text{“xyz”} = \text{“axyz”}\text{, using } \texttt{compl}\ \text{“abcde”} = [\text{'a'}]$$

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Better:

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$$\texttt{shapeInv}\ l_s\ l_v = \texttt{head}\ [n+1 \mid n \leftarrow (l_s - 1) : [0..],\ (\texttt{length}\ (\texttt{get}\ [0..n])) == l_v]$$

Conclusion

[V., POPL'09]:

- very lightweight, easy access to bidirectionality
- full treatment of equality and ordering constraints
- proofs, using free theorems and equational reasoning
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- efficiency issues untackled so far, ...

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